Original sources in the classroom and their educational effects

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HPM 2016, Montpellier

Introduction: scope of the talk

What:

- Corpus: a talk that is rooted in a French community
  Commission inter-IREM Histoire et Épistémologie des mathématiques
- Aims:
  - Discuss recent work
  - Spell out research questions

What not:

- History as a goal
- Relationship between History of maths and Didactics of maths as disciplines and bodies of knowledge
  Cf. Barbin 1997, Fried 2001 & 2007, Chorlay & Hosson 2016

Introduction

What:

- Corpus:

  E. Barbin (ed.) *Let History into the Mathematics Classroom*

  Springer, fall of 2016

- Aims:
  - To characterize the IREM approach to the use of original sources in the classroom
  - On this basis, to spell out research questions as to
    - The reception of such pedagogical documents by teachers
    - The expected / observed educational effects
  - To spell out research questions that are not *absolutely specific* to the topic ‘use of original sources in the classroom’

Outline of the talk

1. *Let History into the Mathematics Classroom*, and reflections thereupon

   1.1 An example: *When Leibniz plays dice*

   1.2 A shared practice

2. Research questions

   2.1 Meta-tasks: delineation, expected educational effects

   2.2 Any demand for what we supply?

3. Ongoing work on M-tasks in primary school

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An example: *When Leibniz plays dice* (1678)

(...) *But in order to make this matter more intelligible, I first of all say that appearance* [probability] *can be estimated, and even that it can be sold or bought.*

(…) *Let us take an example. Two people are playing at dice* [with two dice]*: one will win if he scores eight points again, the other if he gets five. It is a question of knowing which of the two it would be best to bet on. I say that it should be the one who needs eight points, and even that his advantage compared with the hope that the other must have, is three to two. That is to say that I could bet three écus to two for the one who needs eight points against the other without doing myself any harm. And if I bet one against one, I have a great advantage. It is true that notwithstanding the chance, I might lose; especially since the chance of losing is like two and that of winning is like three. But as time goes by, observing these rules of chance, and playing or betting often, it is constant that at the end, I will have won rather than lost.*

But to show that there is a greater probability for the player needing eight points, here is a demonstration. I suppose that they are playing with two dice, and that the two dice are well made, without any cheating. This being the case, it is clear that there are only two ways to reach five points; one is 1 and 4, the other 2 and 3. However there are three ways to score eight points, i.e. 2 and 6, 3 and 5 and also 4 and 4. Now each of these ways has in itself as much probability as the other as, for example, there is no reason to say that there is more probability of getting 1 and 4 than 3 and 5. Consequently, there are as many probabilities (equal amongst themselves) as there are of ways. So if five points can only be made in two ways, but eight points can be made in three ways, it is clear that there are two chances of getting five and three chances of getting eight.

(…) *That being the case, it is obvious that the estimate I have just made is the one to follow. That is to say that this fundamental maxim will be the case:*

The chance or probability of outcome A keeps the same proportion to the chance or probability of outcome B as the number of all the ways capable of producing outcome A has proportionally to all the ways of producing outcome B, supposing all these ways are equally doable.

An example: *When Leibniz plays dice* (1678)

Context of the lesson plan:

- A 2-hour session of guided reading
- High-school students (age 15), with no prior knowledge of probability theory
- Preparatory work: look up in a dictionary the meaning of “heuristic”, “*a priori*”, “*a posteriori*”, “empirical”, “*aléatoire*” (random), and “*hasard*” (chance). For the last two, also check the etymological origin.

An example: *When Leibniz plays dice* (1678)

Outline of lesson plan

Meaning of the "*three to two*" ratio

In terms of relative frequencies: 60% for sum 8, 40% for sum 5

Calculation of mean / average / expected gain

Rational decision in the face of randomness

Where do the values 3 and 2 come from?

"counting all the ways", valid only if the dice are fair

Equally likely outcomes

Values determined *a priori* (by reasoning),

not *a posteriori* (observation, empirical data)

Difference between probability theory and descriptive statistics

An example: *When Leibniz plays dice* (1678)

Back to the first question, with a twist: “what do these values 60%, and 40% mean? Do they enable us to say what will happen at the *next* throw of two fair dice?”

No

“*It is true that notwithstanding the chance that I might lose; especially since the chance of losing is like two and that of winning is like three. But as time goes by, observing these rules of chance, and playing or betting often, it is constant that at the end, I will have won rather than lost.*”

Frequentist approach to probabilities

Informal statement of the law of large numbers

An example: *When Leibniz plays dice* (1678)

Does the law of large number provide a means to check that the values which Leibniz determined by pure reasoning give a correct quantitative description of the random experiment?

Yes. We could use simulation (spreadsheet or *ad hoc* algorithm)

Turns out the estimated probabilities stabilise around 55% - 45%,

instead of 60% - 40%

There must be a flaw in Leibniz's reasoning

What if the two dice were different colours?

Two models for the same random experiment

Case	2 - 6	6 - 2	3 - 5	5 - 3	4 - 4	1 - 4	4 - 1	2 - 3	3 - 2
Probability	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9

Case	2+6=8	3+5=8	4+4=8	1+4=5	2+3=5
Probability	2/9	2/9	1/9	2/9	2/9

An example: *When Leibniz plays dice* (1678)

Curriculum-assigned goals

- Notions: probability, probability distribution, equally likely outcomes, uniform distribution, expected value, fair bet (null expectation)

- Know-how: use a probability distribution to work out the expectation of a random variable; use a simulation to estimate probabilities

- Epistemological insight: connection between descriptive statistics and probability theory; informal statement of the law of large numbers

A shared practice

Let History into the Mathematics Classroom

Edited by Évelyne Barbin

Translated by Janet Tansom, Peter Ransom and Chris Weeks

Springer
the language of science

- Angles in Secondary School: Surveying and Navigation
- Dividing a Triangle in the Middle Ages: An Example From the Latin Works on Practical Geometry
- A Square in a Triangle
- Indian Calculation: The Rule of Three – Quite a Story…
- The Arithmetic of Juan de Ortega: Equations without Algebra
- The Congruence Machine of the Carissan Brothers
- A Graphic Approach to Euler's Method
- Calculating with Hyperbolas and Parabolas
- When Leibniz Plays Dice
- The Probability of Causes According to Condorcet

A shared practice

- The starting point for the design of a lesson plan is usually the recognition of the fact that some local teaching need – grounded in the curriculum – is somewhat echoed in a specific historical source.

- As far as timing is concerned, we're talking medium range: beyond 10-min exercises, yet not complete chapter designs. Enrichment of teaching sequences rather than reconstruction.

- A gradient of explicitness
    - On the one hand: explicit cognitive and conceptual goals;
      HM a means, not a goal.
    - On the other hand: issues such as motivation, image of mathematics, or the transmission of historical knowledge are kept in the background, or altogether absent.

A shared practice

- The tasks that are entrusted to the students are usually rather demanding, even difficult. A fact that the designer does fully acknowledge.

- The chapters are meant to be used as ressources for other teachers. Consequently, one has to distinguish between two intended audiences :
  - Secondary school students
  - Teachers and teacher-trainers

- The primary source in selected primarily for its didactical potential, not for its historical value.

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3. Ongoing work on M-tasks in primary school

Some research questions

A definite teacher – student gradient of explicitness in the IREM publications

- Teacher motivation
- Background knowledge: historical, mathematical
- Primary sources

- Tasks entrusted to student
- Tasks actually performed

- Expected educational effects
- Actual educational effects

Some research questions

A definite teacher – student gradient of explicitness in the IREM publications

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- Actual educational effects

Some research questions

Focusing on tasks: starting from a small sample

“In each case there were issues around the understanding of the texts and checking the proofs, even of completing or adding to them.”

(Guyot, on inscribing a square in a triangle)

“Summarize the solution and explain the method. (…) Is it true or false? (…) The translator made a mistake in the problem. What is the incorrect word and what word should replace it? (...) Transcribe the problem in French everyone can understand (take care with the wording). Represent the situation with a simple diagram”

(Métin, on false position)

Some research questions

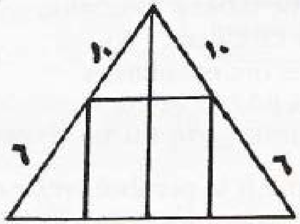
Focusing on tasks: another sample

Calculer à la manière d'Al-Khwārizmi

Dans un traité du IX^{e} siècle, on trouve le problème suivant : « Dans un triangle isocèle de base 12 coudées, on trace un terrain carré. Quel est son côté ? ».

1. L'auteur du traité, Al-Khwàrizmí, nous dit : « Nous considérons un des côtés du terrain carré égal à une *chose* et nous la multiplions par elle-même ; il vient un *bien*. [...] ». Que représente une *chose* sur la figure ? Et un *bien* ?

نصف شيء فيكون ستة أشياء إلا نصف مال وهو تكسير المثلثين جميعاً اللتين
هما على جنبتي المربعة . فأما تكسير المثلثة العليا فهو أن تضرب ثمانية غير شيء
وهو العمود في نصف شيء فيكون أربعة أشياء إلا نصف مال فهذا هو تكسير
المربعة وتكسير الثلاث مثلثات وهو
عشرة أشياء تعدل ثمانية وأربعين هو
تكسير المثلثة العظمى فالشيء الواحد من
ذلك أربعة أذرع وأربعة أخماس ذراع
وهو كل جانب من المربعـــة وهـذه
صورتهـا .



2. Al-Khwàrizmí nous donne ensuite le calcul suivant :
« Quant aux deux triangles qui sont sur les flancs [...] leur aire est que tu multiplies une *chose* par six moins un demi d'une *chose*, il vient six *choses* moins la moitié d'un *bien*. »
Expliquer ce calcul.

3. Avec nos notations actuelles, si on note ℓ une *chose*, comment s'écrit un *bien* ?
Écrire le calcul précédent avec ces notations.

Some research questions

Beyond verbs, what tasks?

- Text-reading tasks which are *not* specific to mathematical texts

- Meta-tasks (M-tasks) in mathematics

Some research questions

Delineating the class of M-tasks (1)

- they differ from directly transformative tasks (such as: draw ..., work out ..., factorize ..., solve equation ...)
- students are required to do something *with reference to*, or *about* a piece of mathematics; something which is not limited to acting *on* or *within* the mathematics involved
- non-routine tasks; tasks which are not necessarily meant to ever become routine tasks
- to carry out these tasks, the local teaching context usually provides no available standardized technique nor background technology (to use ATD terminology)

Some research questions

Delineating the class of M-tasks (2)

- by their very nature, such Meta-tasks involve the production of technological discourse by the students
- students are to reformulate, and assess some mathematical content on the basis of the maths they have learnt, thus acting as *experts* endowed with background knowledge
- relevant background mathematical knowledge is usually varied, and identifying the relevant background knowledge is demanding in itself

Some research questions

M-tasks: question #1

Verbs such as:

justify, compare, assess, criticize, summarize, prove,
reformulate/translate/rewrite

do not always point to M-tasks. Under what conditions do they actually do? Is the absence of a well-identified local background technique a sufficient condition?

M-tasks: question #2

What is the role of *proof*-tasks among M-tasks?
More specifically: are M-tasks other than “proving” conducive to “proving”, from didactical and currilar perspectives ?

Some research questions

M-tasks: question #3
What are the expected educational effects?

Promising leads:

- triggering “commognitive” conflict (i.e. conflict as to meta-rules)
  See Kjeldsen, HPM 2012

- fostering cognitive flexibility
  More precisely: transition from *mobilizable* knowledge to *available* knowledge (to use the terminology of Aline Robert)
  Some knowledge (or know-how) is mobilizable if students can apply it reasonably successfully upon request (e.g. “Use Pythagoras’ rule to work out the length of side AC”); it has become available when students are able to identify it as the relevant tool even when no indications are given

... insert here your favourite concept (when it comes to capturing what “cognitive flexibility” means) ...

Some research questions

M-tasks: question #4

Are M-tasks specific to the use of original sources? Answer: No

A lead: a comparison with “open problems” (*problèmes ouverts*)

Characteristics of a *problèmes ouverts*:

- A short question
- No hints as to the answer
- No hints as to the method or the steps
- Situations in which it is easy to engage in conjectures, trials etc.
- Situations/milieus which send enough feedback for students to be able to assess by themselves the validity of their conjectures and trials
- The final solution requires more than one move, and more than one simple technique

Some research questions

Sessions based on primary historical sources / *problèmes ouverts* (ct'd)

Dissimilarities: … too many to be listed

Similarities:

- relevant background mathematical knowledge is usually varied, and identifying the relevant background knowledge is demanding in itself
- in didactical research: identifying the *intended* educational effects remains difficult, if one is not content with general notions such as "cognitive flexibility", and "methodological skills"
- in didactical research: identifying the *actual* education effects remains difficult, all the more since these such sessions are usually rare, and the intended effects hard to pinpoint
- time-consuming

Some research questions

Sessions based on primary historical sources

Besides *problèmes ouverts*, other possible comparisons

- with exercises which require the assessment – by students – of mathematical statements, while providing little background support

  True or False. Justify your answer
  True / False / No way to know. Justify your answer

- with the teaching and research protocol of *débat scientifique* (scientific debate in the classroom)

Some research questions

M-tasks: series of question #5

Turning to teachers, is there any demand for what we supply?

Hypotheses: the fact that what we supply is M-task-rich

- is easily spotted by teachers and teacher-trainers
- may account, to a large extent, for the *reluctance* of some (many?) to engage in such sessions, whatever the quality of the documents we supply

To test the second hypothesis, we need to find ways to assess the relative weights of “reluctance factors”, among which: lack of historical knowledge, lack of curriculum support etc.

A strong correlation, in the selection of classroom activities by teachers, between interest in *problèmes ouverts* and interest for sessions using historical sources would support this hypothesis

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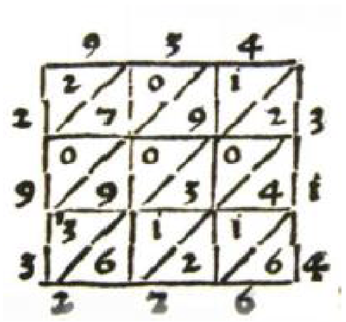
On-going work on M-tasks in primary school

Design of a three-session teaching sequence

Experiment carried out in the fall of 2015, in four classes of the final year of primary school, in France

Starting from documents such as these two:



+ a video of someone performing *one* such multiplication, namely 93×52

Ongoing work on M-tasks in primary school

What not:

- HM as a goal
- Improve multiplication skills / teach a new technique
- Investigate the validity of this technique, leading to reflections on place value, or the distributivity of × over +

What: explore the extent to which some M-tasks could be entrusted to young students in an *algorithmic* context

Background motivation:

- New emphasis on algorithmic thinking in the French maths curriculum
- Long-term historical research on algorithmic texts in ancient mathematics at the SPHere research team (Uni. Paris Diderot), with an emphasis on the expression of *generality*

  cf. K. Chemla, R. Chorlay, D. Rabouin (eds) *Oxford Handbook of Generality in Mathematics and the Sciences*, OUP 2016

Ongoing work on M-tasks in primary school

Standard tasks in an algorithmic context (usually at a higher level)

- Make a conjecture as to the function of a given algorithm
  Prove/disprove it
- Write an algorithm performing a specific function
- Modify a given algorithm for a given purpose
- Prove correctness
- Prove termination (when relevant)
- Compare two algorithms in terms of complexity

Ongoing work on M-tasks in primary school

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Ongoing work on M-tasks in primary school

Write an algorithm performing a specific function

Assignment: *On paper, explain the* gelosia *method for students with no access to the video. They should be able to apply your method starting from any two whole numbers.*

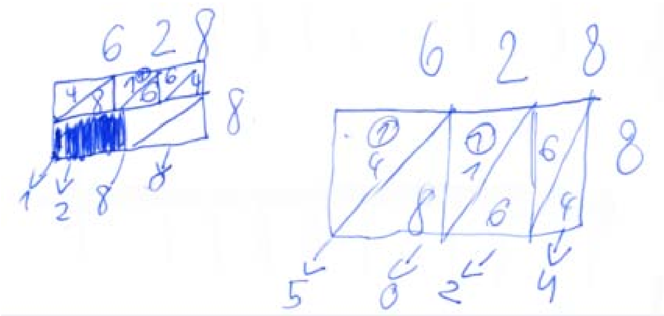
This involves:

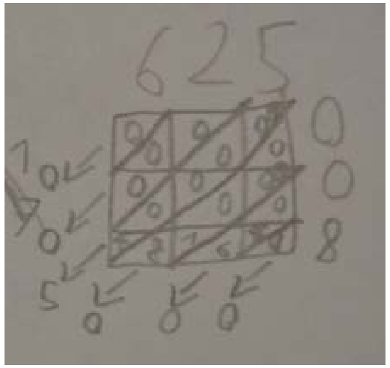
- Identifying the steps and their correct ordering
- Finding semiotic means to express the steps
- Meeting the challenge of *generality*

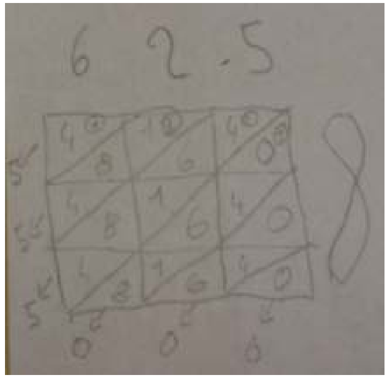
Ongoing work on M-tasks in primary school

Facing the challenge of generality

When carrying out the technique: how to alter the technique in order to pass from 93 × 52 to 625 × 8





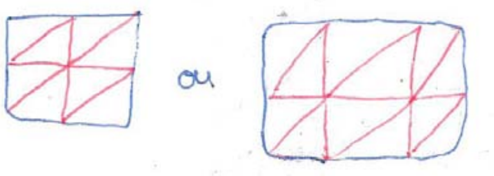


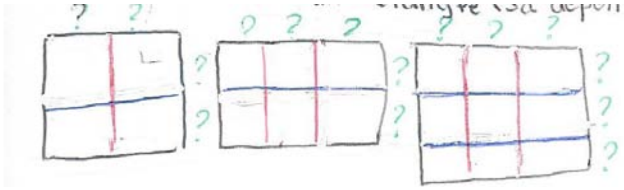
Ongoing work on M-tasks in primary school

Facing the challenge of generality

When writing the algorithm for “any number”

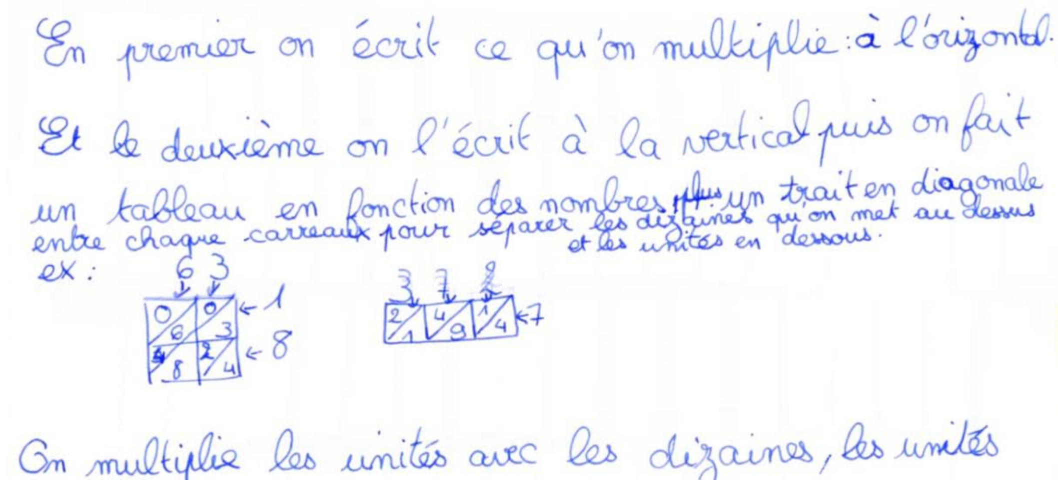
Option A: using a generic example, then pointing to the size-variability





Option B:
inserting non-generic examples in a rhetorical layout

See video S2-Rouen3 12’50 – 14’


En premier on écrit ce qu'on multiplie : à l'orizontal.
Et le deuxième on l'écrit à la vertical puis on fait un tableau en fonction des nombres plus un trait en diagonale entre chaque carreaux pour séparer les dizaines qu'on met au dessus et les unités en dessous.
ex :

On multiplie les unités avec les dizaines, les unités

Ongoing work on M-tasks in primary school

Compare two algorithms in terms of complexity

Assignment: *If you were to recommend one of the two techniques for another class, which one would you choose, and why?*

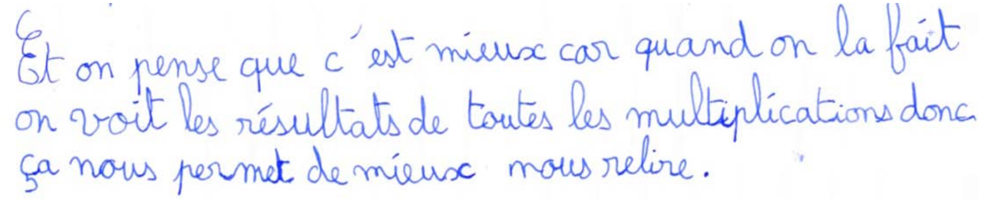
Ongoing work on M-tasks in primary school

Compare two algorithms in terms of complexity

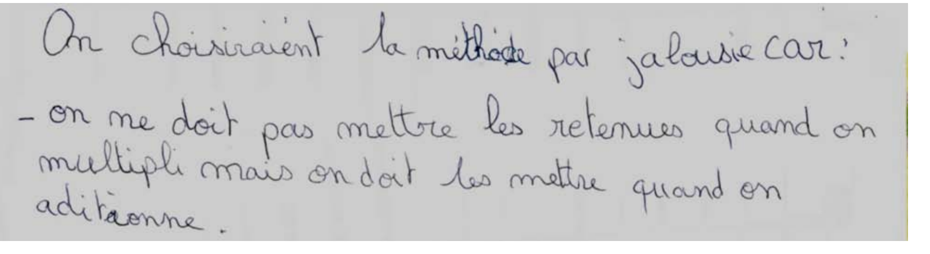
Request: *If you were to recommend one of the two techniques for another class, which one would you choose, and why?*

What students think of when working on their own: ergonomic aspects

- *Gelosia* technique more surveyable


Et on pense que c'est mieux car quand on la fait on voit les résultats de toutes les multiplications donc ça nous permet de mieux nous relire.

- No carries in the multiplicative phase of the *gelosia* technique

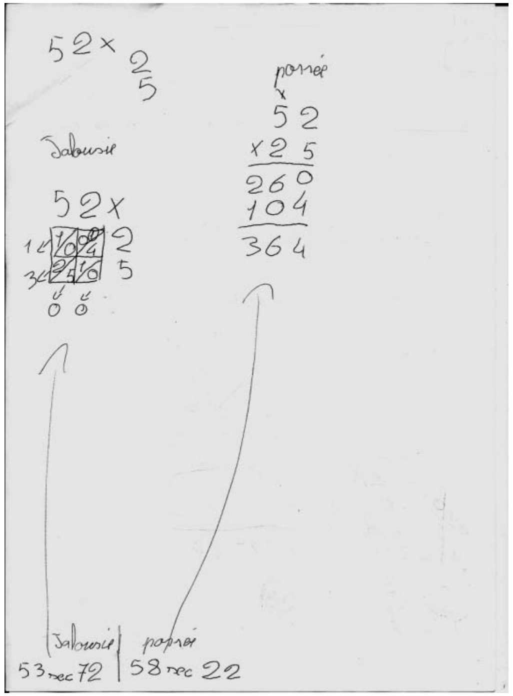

On choisiraient la méthode par jalousie car :
- on me doit pas mettre les retenues quand on multipli mais on doit les mettre quand on aditionne.

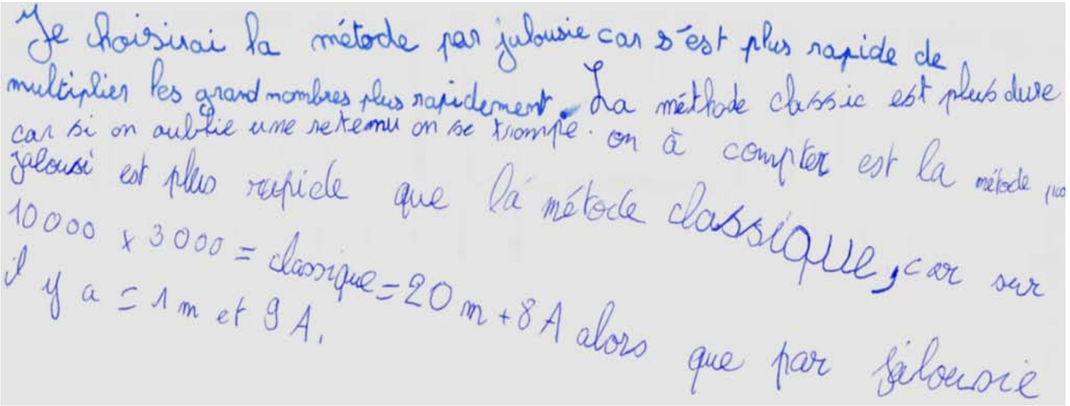
- No need to write extra zeroes when multiplying by a several-digit number

Ongoing work on M-tasks in primary school

Compare two algorithms in terms of complexity

What students engage in when prompted to explain what they mean when they say one is “faster” than the other:





Conclusion - Perspectives

Looking back on the primary school experiment:

- In well-defined contexts, young students can successfully engage in M-tasks such as: to formulate a *general* method ; to compare two algorithms (and find criteria for this comparison)

- Left in the background :
  - Intended educational effects? What did they *learn*?
  - Can we justify the fact that we decided that the proof of correctness of the *gelosia* method could not be entrusted to students working in autonomous groups? More generally: what are the conditions for this type of session to work?

Conclusion - Perspectives

Next moves ?

- Try to answer (or refine) one of the 5 above-listed research questions

- Learn from a *failed* experiment of using historical sources in the classroom cf. doctoral dissertation of Charlotte de Varent (to be defended in 2017)

- A new editorial project for the *Commission inter-IREM*: historical sources for the "*cycle 3*" (last 2 years of primary school + 1st year of middle school)

- Work on *reading tasks* in collaboration with didacticians of literacy

Thank you for your attention

Merci pour votre attention