***Last update*: 15 June 2016**

History of Mathematics in Mathematics Education: Recent developments

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Key words: History & Pedagogy of Mathematics (HPM); History of Mathematics; Epistemology of Mathematics; HPM perspective; HPM domain; HPM group; Original sources; Theoretical frameworks & constructs; Interdisciplinary teaching; Teacher education; Cultures & Mathematics; Bibliographical survey.

Abstract: This is a survey on the recent developments (since 2000) concerning research on the relations between History and Pedagogy of Mathematics (the *HPM domain*). Section 1 explains the rationale of the study and formulates the key issues. Section 2 gives a brief historical account of the development of the *HPM domain* with focus on the main activities in its context and their outcomes. Section 3 provides a sufficiently comprehensive bibliographical survey of the work done in this area since 2000. Finally, section 4 summarizes the main points of this study.

1. Introduction

This is a survey describing the state-of-the-art on the themes of the ICME 13 *Topic Study Group* (TSG) 25: ***The Role of History of Mathematics in Mathematics Education***. It gives a brief account of the developments since 2000[1] on the relations between History and Pedagogy of Mathematics, in order to illuminate and provide insights on the following general questions:

- *Which history is suitable, pertinent, and relevant to Mathematics Education (ME)?*
- *Which role can History of Mathematics (HM) play in ME?*
- *To what extent has HM been integrated in ME (curricula, textbooks, educational aids/resource material, teacher education)?*
- *How can this role be evaluated and assessed and to what extent does it contribute to the teaching and learning of mathematics?*

These are the key issues explicitly addressed in and/or implicitly underlying what we call the ***HPM perspective*** as detailed below.

1.1 The HPM Perspective

Mathematics is a human intellectual enterprise with a long history and a vivid present. Thus,

[1] The year of publication of *History in Mathematics Education: The ICMI Study*, edited by J. Fauvel & J. van Maanen (Kluwer 2000); a highly collective work that has been a landmark in the area (see section 2, p.4).

mathematical knowledge is determined not only by the circumstances in which it becomes a deductively structured theory, but also by the procedures that originally led or may lead to it. Learning mathematics includes not only the "polished products" of mathematical activity, but also the understanding of implicit motivations, the sense-making actions and the reflective processes of mathematicians, which aim to the construction of meaning. Teaching mathematics should give to students the opportunity to "do mathematics." In other words, although the "polished products" of mathematics form the part of mathematical knowledge that is communicated, criticized (in order to be finally accepted or rejected), and serve as the basis for new work, the **process** of producing mathematical knowledge is equally important, especially from a didactical point of view. Perceiving mathematics both as a logically structured collection of intellectual products and as processes of knowledge production should be the core of the teaching of mathematics. At the same time, it should be also central to the image of mathematics communicated to the outside world.

Along these lines, putting emphasis on integrating historical and epistemological issues in mathematics teaching and learning constitutes a possible natural way for exposing mathematics in the making that may lead to a better understanding of specific parts of mathematics and to a deeper awareness of what mathematics as a discipline is. This is important for ME, helping to realize that mathematics:

- is the result of contributions from many different cultures;
- has been in constant dialogue with other scientific disciplines, philosophy, the arts and technology;
- has undergone changes over time; there have been shifting views of what mathematics is;
- has constituted a constant force for stimulating and supporting scientific, technical, artistic and social development.

This helps to improve ME at all levels and to realize that although mathematics is central to our modern society and a mathematically literate citizenry is essential to a country's vitality, historical and epistemological issues of mathematics are equally important. The harmony of mathematics with other intellectual and cultural pursuits also makes the subject interesting, meaningful, and worthwhile. In this wider context, history and epistemology of mathematics have an additional important role to play in providing a fuller education of the community: not being a natural science, but a formal science closer to logic – hence to philosophy – mathematics has the ability inherent in itself to connect the humanities with the sciences. Now that societies value and want young people educated in the sciences, but have a hard time finding out how to get people to "move" from humanities to the sciences, integrating history and epistemology in ME can make this connection visible to students. This is most important, especially today when there is much concern about the level of mathematics that students are learning and about their decreasing interest in mathematics, at a time when the need for both technical skills and a broader education is rising.

1.2 Summary of the content of the present survey

The rationale underlying this perspective has formed the core and main concern of the approaches adopted towards integrating History and Epistemology of Mathematics in ME (the ***HPM domain***), especially in the context of the ICMI affiliated *International Study Group on the relations between the History and Pedagogy of Mathematics* (the ***HPM Group***) since its formation in 1972.

What follows consists of four sections:

Section 2 gives a brief historical account of the development of the *HPM domain* with focus

on the main activities in its context and their outcomes since 2000 (§2.1); a short presentation of journals and newsletters (§2.2); and an outline with comments on the key issues mentioned in section 1 and references to the literature for details (§2.3).

Section 3 constitutes the major part of the survey. It provides a sufficiently comprehensive bibliographical survey of the work done since 2000: Collective works in §3.1 (collective volumes, special ME journal issues, conference proceedings, resource material); individual works in §3.2 (books & doctoral dissertations, papers in scientific journals, collective volumes and conference proceedings).

Though the emphasis is on research results of an as broad as possible international interest, due attention is paid to nationally-oriented implementation of the *HPM perspective* as well.

Remarks: (a) Next to each reference the TSG 25 themes to which it is related are indicated, numbered as in the Appendix below.

(b) Collective works exclusively on the *HPM perspective* (i.e. those in §3.1) contain several important contributions. However, in order to keep this survey to a reasonable size, these contributions are note included as separate items in §3.2, though some of them are quoted in section 2 (hence, they appear in section 5). Instead, these collective works are annotated briefly.

Section 4 summarizes the main points of this study.

Section 5 contains all references given in section 2.

Important remarks: To keep this document - especially the bibliographical survey - to a reasonable size, (a) several **abbreviations** are used for the titles of journals, conferences etc, all explained at the beginning of **section 3**; (b) many lengthy URL have been integrated into the title of the reference to avoid making the text difficult to follow and non-appealing. These URLs are **not** displayed in a print-out of this document, so the reader is advised to use its electronic version available at http://www.clab.edc.uoc.gr/HPM/HPMinME-TopicalStudy-18-2-16-NewsletterVersion.pdf

For several references (especially those in the annotated bibliography in section 3.1), we provide hyperlinks, where one can find online additional information in the form of an abstract, review, outline of contents etc.

APPENDIX: *Main Themes of TSG 25*

T1: Theoretical and/or conceptual frameworks for integrating history in ME.

T2: History and epistemology implemented in ME, considered from either the cognitive or/and affective points of view:

a. Classroom experiments at school, the university and teacher pre- & in-service education;
b. Teaching material: textbooks, resource material of any kind.

T3: Surveys on:

a. Research on the HM in ME;
b. The HM as it appears in curriculum and/or textbooks.

T4: Original sources in the classroom and their educational effects.

T5: History and epistemology as a tool for an interdisciplinary approach in the teaching and learning of mathematics and the sciences by unfolding their fruitful interrelations.

T6: Cultures and mathematics fruitfully interwoven.

2. An outline of the historical development of the HPM domain

Integrating HM in ME has been advocated since the second half of the 19th century, when mathematicians like De Morgan, Poincaré, Klein and others explicitly supported this path and historians like Tannery and later Loria showed an active interest on the role HM can play in

ME. At the beginning of the 20th century, this interest was revived as a consequence of the debates on the foundations of mathematics. Later on, history became a resource for various epistemological approaches; Bachelard's historical epistemology, Piaget's genetic epistemology and Freudenthal's phenomenological epistemology, at the same time stimulating the formulation of specific ideas and conclusions on the learning process (Fauvel & van Maanen 2000, p.202; Barbin & Tzanakis 2014, p.256 and references therein).

This interest became stronger and more competitive in the period 1960-1980 in response to the *New Math* reform, when its proponents were strongly against "a historical conception of ME," whereas for its critics, HM appeared like a "therapy against dogmatism", conceiving mathematics not only as a language, but also as a human activity. In 1969, the NCTM in USA devoted its 31st Yearbook to the HM as a teaching tool (NCTM 1969) and in the 1970s a widespread international movement began to take shape, greatly stimulated and supported by the establishment of the *HPM Group* at ICME 2 in 1972 and its scope in 1978 (HPM Group 1978).

Thus, during the last 40 years, integrating HM in ME has evolved into a worldwide intensively studied area of new pedagogical practices and specific research activities and a gradually increasing awareness has emerged of what was described in §1.1 as the *HPM perspective* (Fasanelli & Fauvel 2006 for a historical account and references prior to 2000; Furinghetti 2012, Barbin 2013, Barbin & Tzanakis 2014 for a concise outline of later developments, and references there in).

The rising international interest in the *HPM perspective* and the various activities of the HPM Group worldwide, led to the approval by ICMI in 1996 of launching a 4-year *ICMI Study* on the relations between HM and ME. After a *Discussion Document* drawn up by the *Study* co-chairs (Fauvel & van Maanen 1997) and a *Study Conference* in 1998, at Luminy, France, the Study culminated in the publication of a 437-page volume written by 62 contributors working together in 11 groups (Fauvel & van Maanen 2000). This was a landmark in establishing and making more widely visible the *HPM perspective* as a research domain in the context of ME and greatly stimulated and enhanced the international interest of the educational community in this area.

Below we give an account of the main regular activities and their outcomes concerning educational research and its implementation in educational practice, relevant to the *HPM domain* and mainly realized in the context of the *HPM Group*.

2.1 Meetings and related collective volumes

2.1.1 *ICME Satellite Meetings of the HPM Group*

These quadrennial meetings are a major activity to bring together individuals with a keen interest in the relationship between the HM and ME; researchers in ME interested in the HM in relation to mathematical thinking, mathematics teachers at all levels eager to gain insights into the *HPM perspective*, historians of mathematics wishing to talk about their research, and mathematicians wanting to learn about new possibilities to teach their discipline. and all those with an interest in the *HPM domain*.

They are organized just after, or before the ICME:

1984 Adelaide; *ICME 5*, Adelaide
1988 Florence; *ICME 6*, Budapest
1992 Toronto; *ICME 7*, Quebec
1996 Braga, *HEM Braga 96* conjointly with the *2nd ESU*; *ICME 8*, Seville

2000 Taipei, *HPM* 2000; *ICME 9*, Tokyo-Makuhari
2004 Uppsala, *HPM 2004*, conjointly with the *4th ESU*; *ICME 10*, Copenhagen
2008 Mexico City, *HPM* 2008; *ICME 11*, Monterrey
2012 Daejeon, *HPM 2012*; *ICME 12* Seoul
2016 Montpellier, *HPM 2016*; *ICME 13* Hamburg

The books published as a result of these *HPM* meetings are listed below using the abbreviations introduced in section 3:

Swetz *et al* 1995 (after ICME-6); Calinger 1996 (after HPM 1992); Lagarto *et al* 1996 (during HPM 1996); Katz 2000 (after HPM 1996); Horng & Lin 2000 (at HPM 2000); Bekken & Mosvold 2003 (before ICME 10 & HPM 2004); Horng *et al* 2004 (before HPM 2004); Furinghetti *et al* 2004 (at HPM 2004; revised edition Furinghetti *et al* 2006; see §3.1.3); Cantoral *et al* (at HPM 2008); Barbin *et al* 2012 (at HPM 2012; revised edition in progress).

2.1.2. The *European Summer University on the History and Epistemology in Mathematics Education* (ESU)

The initiative of organizing a *Summer University* (SU) on the *History and Epistemology in Mathematics Education* belongs to the French ME community in the early 1980s. The French IREMs organized the first interdisciplinary in 1984, in Le Mans, France, followed by another three in France. The next one was organized in 1993 on a European scale; the 1st *European Summer University on the History and Epistemology in Mathematics Education*, (a name coined since then, abbreviated as ESU since 2004), though many participants come from outside Europe. Since 2010, ESU is organized every four years to avoid coincidence with the HPM meetings.

Since its original conception, ESU has been developed and established into one of the major activities in the HPM domain. It mainly aims to: provide a school for working on a historical, epistemological and cultural approach to mathematics and its teaching, with emphasis on actual implementation; give the opportunity to mathematics teachers, educators and researchers to share their teaching ideas and classroom experience related to a historical perspective in teaching; and motivate further collaboration along these lines among teachers of mathematics and researchers on the HM and ME in Europe and beyond, attempting to reveal and strengthen the *HPM perspective*. Below is a list of the ESUs:

1993, *ESU* 1 Montpellier
1996, *ESU* 2 Braga (conjointly with *HEM Braga 96*)
1999, *ESU* 3 Leuven & Louvain-la-Neuve
2004, *ESU* 4 Uppsala (conjointly with *HPM 2004*)
2007, *ESU* 5 Prague
2010, *ESU* 6 Vienna
2014, *ESU* 7 Copenhagen

The following works were published after the *ESU*s: Lalande *et al* 1995; Lagarto *et al* 1996; Radelet-de-Grave & Brichard 2001; Furinghetti *et al* 2004; Barbin *et al* 2008; Barbin *et al* 2011a; Barbin *et al* 2015.

2.1.3. The *HPM domain* at ICMEs

Activities related to the *HPM perspective* have always been present in the ICMEs; Fasanelli & Fauvel 2006 for ICMEs before 2000. Since 2000, such activities have formed an established part of the ICMEs' scientific program:

(a) *ICME 9, Tokyo*, Japan, 2000

WG for Action 13: *History and Culture in Mathematics Education* coordinated by J. van Maanen, W.S. Horng.

Summary in H. Fujita, Y. Hashimoto, B.R. Hodgson, P.Y. Lee, S. Lerman & T. Sawada (Eds) (2004). *Proc. of the 9th ICME* (pp.287-291). Dordrecht: Kluwer Academic Publishers, in CD (accessed 18/2/16)

(b) *ICME-10, Copenhagen*, Denmark, 2004

TSG 17: *The role of the history of mathematics in mathematics education*, organized by M.K. Siu, C. Tzanakis, A. El Idrissi, S. Kaisjer, L. Radford.

Summary in M. Niss (Ed) (2008). *Proc. of the 10th ICME* (pp.363-367). IMFUFA, Roskilde University, in CD (accessed 18/2/16)

This TSG led to a post-conference publication (Siu & Tzanakis 2004)

(c) *ICME 11 Monterrey*, Mexico, 2008

TSG 23: *The role of the history of mathematics in mathematics education*, organized by A. El Idrissi, A. Miguel, F. Furinghetti, A. Garciadiego, E. Barbin (accessed 18/2/16).

(d) *ICME 12 Seoul*, Korea, 2012

TSG 20: *The role of history of mathematics in mathematics education*, organized by R. Chorlay, W-S. Horng, M. Kronfellner, K. Clark, A. El Idrissi, H. Chang (accessed 18/2/16).

Summary in S. J. Cho (Ed) (2015). *Proc. of the 12th ICME: Intellectual and attitudinal challenges*. New York: Springer (pp.485-487) http://www.icme12.org/sub/sub02_05.asp
12 papers & 13 posters available at
http://www.icme12.org/sub/tsg/tsg_last_view.asp?tsg_param=20 (accessed 18/2/16).

(e) *ICME 13 Hamburg*, Germany 2016

TSG 25*:* *The role of history of mathematics in mathematics education*, organized by C. Tzanakis, X. Wang, K. Clark, T.H. Kjeldsen, S. Schorcht (accessed 4/4/2016)

2.1.4. The *HPM domain* at CERME

CERME is a regular activity of the *European Society for Research in Mathematics Education* (ERME), organized every two years in the form of presentations, discussions and debates within thematic WG. Though relatively new, the *HPM perspective* has exhibited great potential at CERME and is expected to play a central role in the future:

(a) *CERME 6, Lyon*, France, 2009

WG 15: *Theory and research on the role of history in mathematics education*, organized by F. Furinghetti, J-L. Dorier, U. T. Jankvist, J. van Maanen & C. Tzanakis
A new WG structured along 7 themes; 13 papers & 1 posteraccepted and included in the proceedings (see §3.1.3).

(b) *CERME 7*, Rzeszów, Poland, 2011

WG 12: *History in Mathematics Education* organized by U. T. Jankvist, S. Lawrence, C. Tzanakis & J. van Maanen.

Structured along 9 themes; 13 papers & 1 poster accepted and included in the proceedings (see §3.1.3).

(c) *CERME 8*, Antalya, Turkey, 2013

WG 12: *History in Mathematics Education* organized by U. T. Jankvist, K. Clark, S. Lawrence & J. van Maanen.

Structured along 9 themes (the same as CERME 7); 12 papers & 3 posters accepted and included in the proceedings (see §3.1.3)

(d) *CERME 9, Prague*, Czech Republic, 2015

Thematic WG 12: *History in Mathematics Education*, organized by R. Chorlay, U. T. Jankvist, K. Clark & J. van Maanen (accessed 15/6/2016).

The WG themes were modified considerably, becoming more specific. This reflects further deepening of research in this area, with emphasis both on empirical work and its assessment and on sharpening theoretical ideas and developing conceptual frameworks adequate for describing and understanding phenomena relevant to the *HPM perspective* (see §3.1.3).

(d) *CERME 10, Dublin*, Ireland, 2017

Thematic WG 12: *History in Mathematics Education*, organized by R. Chorlay, S. Lawrence, K. Clark & Katalin Gosztonyi (accessed 15/6/2016).

Structured along 4 main themes with focus on empirical studies, surveys of existing uses of HM in ME, and relationships between such empirical studies and theories, frameworks and studies in other parts of ME research.

2.2 Journals and Newsletters

2.2.1. *Convergence: Where Mathematics, History and Teaching Interact*

Since 2004, the MAA has published *Convergence*: *Where Mathematics, History and Teaching Interact*, a free online journal in HM and its use in teaching.

Aimed at teachers of mathematics at both the secondary and collegiate levels, *Convergence* includes topics from grades 8-16 mathematics, with special emphasis on grades 8-14. Its resources for using the HM in mathematics teaching include informative articles about the HM, translations of original sources, classroom activities, projects and modules, teaching tools such as its *Mathematical Treasures*, reviews of new and old books, websites, *Problems from another time*, and other teaching aids that focus on utility in the classroom.

2.2.2. The *Bulletin of the British Society for the History of Mathematics (BSHM Bulletin)*

The *BSHM Bulletin* aims to promote research into the HM and to encourage its use at all levels of ME. Articles on local HM and the use of HM in ME are particularly encouraged. Originally published as a Newsletter, until 2004 when its 50th issue became Bulletin 1. Under the influence of the late J. Fauvel, president of BSHM (1992–94), editor of its Newsletter (1995–2001) and chair of the HPM Group (1992-96) and his successor, the late J. Stedall, the Newsletter changed from providing information to members into a scientific journal with a regular Education Section directly related to issues relevant to the HPM perspective since 2002 (issue No 46).

2.2.3. The *HPM Newsletter*

It appears 3 times per year since 1980. Originally it was available by contacting the regional distributors; however, for the last 13 years it is also available online from the HPM Group website and its *Newsletter* webpage

It includes a calendar of upcoming events, a guest editorial, a '*Have You Read?*' section, short reviews and announcements of meetings and activities. Furthermore, for the last 13 years it has also included short articles, reports on research projects and PhD theses, book reviews, lists of relevant websites, particular themes that are suggested for further research.

2.3 Comments on some key issues

From what has been presented so far and will become clearer from the bibliographical

survey in Section 3, the last two decades have generated considerable research activity related to the *HPM perspective* of great variety: doing empirical research based on actual classroom implementations; designing specific teaching units; developing various kinds of teaching aids; exploring and understanding students' response to the introduction of the HM in teaching (including teacher education); designing, applying and evaluating interdisciplinary teaching; drawing and/or criticising parallels between the historical development and learning in a modern classroom;[2] mutually profiting from theoretical constructs and conceptual frameworks developed in the context of other disciplines, especially philosophy, epistemology and cognitive science; and evaluating the effectiveness of all this in practice.

The key issues mentioned in Section 1 permeate all these activities as recurring themes that form the *leitmotif* of the HPM domain. Though impossible to present all the work done and the not always mutually compatible opinions and results, below a few general ideas are outlined with reference to the literature for details.

Whether HM is appropriate, or even relevant at all to the teaching and/or learning of mathematics, is an issue that, despite the extended research and the many insightful and sophisticated applications in the last few decades, has not reached universal acceptance even today. In fact, a number of objections against the *HPM perspective* have been raised (Tzanakis, Arcavi *et al* 2000, p.203; Siu 2006; Tzanakis & Thomaidis 2012, §3.4; Furinghetti 2012, §7):

A *Objections of an epistemological and methodological nature*

(a) *On the nature of mathematics*

1. This is not mathematics! Teach the subject first; then its history.
2. Progress in mathematics is to make difficult problems routine, so why bother to look back?
3. What really happened can be rather tortuous. Telling it as it was can confuse rather than enlighten!

(b) *On the difficulties inherent to this approach*

1. Does it really help to read original texts, which is a very difficult and time-consuming task?
2. Is it liable to breed cultural chauvinism and parochial nationalism?
3. Students may have an erratic historical sense of the past that makes historical contextualization of mathematics impossible without having a broader education in general history.

B *Objections of a practical and didactical nature*

(a) *The background and attitude of teachers*

1. Lack of didactical time: no time for it in class!
2. Teachers should be well educated in history: "I am not a professional historian of mathematics. How can I be sure of the exposition's accuracy?"
3. Lack of teacher training.
4. Lack of appropriate didactical and resource material.

(b) *The background and attitude of the students*

1. They regard it as history and they dislike history class!
2. They regard it just as boring as mathematics itself.
3. They do not have enough general knowledge of culture to appreciate it.

(c) *Assessment issues*

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[2] The old but still discussed issue of "historical parallelism" – if and to what extent "ontogenesis recapitulates (aspects of) phylogenesis"; e.g. Radford et al. 2000; Schubring 2006, 2011; Thomaidis & Tzanakis 2007.

1. How can you set questions on it in a test or exam?
2. Is there any empirical evidence that students learn better when HM is made use of in the classroom?

Each of these objections addresses one or more of the four key issues mentioned in Section 1. Below we comment briefly on them in the light of these objections.

2.3.1 *Which history is suitable, pertinent and relevant to Mathematics Education?*

This has been a permanent issue of debate among historians and mathematics educators with an interest in the HPM perspective. As early as 1984 at ICME 5, d'Ambrosio stressed the need to develop three separate histories of mathematics: history as taught in schools, history as developed through the creation of mathematics, and the history of that mathematics which is used in the street and the workplace. To deal with these differences he introduced the concept of *ethnomathematics* as compared to *learned mathematics* (Booker 1986).

In fact, implicit to the objections A(a1), A(a2), A(b1) is the idea that the term "history" is the same, whether used by historians, mathematicians, or teachers and mathematics educators. That this is not so lies at the heart of Grattan-Guinness' early refutation of some of these arguments (Grattan-Guinness 1973; see also Kjeldsen 2011a, pp.1700-1701; 2011b, pp.166-167; Kjeldsen & Blomhøj 2012, §3, and references therein for a different recent approach). On the other hand, it is undeniable that quite often the historical development was complicated, followed a zig-zag path, led to dead ends, included notions, methods and problems that are no longer used in mathematics as we know and work with today, etc. (A(a2), A(b3)). Thus, its integration in ME, on the one hand is nontrivial, and on the other hand poses the question why it must be done at all. Therefore, integrating HM in the teaching and learning of mathematics, may force history "*…to serve aims not only foreign to its own but even antithetical to them*" (Fried 2011, p.13). In other words, the danger of either unacceptably simplifying or/and distorting history to serve education as still another of its tools is real by adopting what has been called a "Whig" (approach to) history, in which "*…the present is the measure of the past. Hence, what one considers significant in history is precisely what leads to something deemed significant today*" (Fried 2001, p.395).

In this connection, an important step was Grattan-Guiness' distinction between what he called *History* and *Heritage* trying to clarify existing conflicts and tensions between a mathematician's and a historian's approach to mathematical knowledge, and paying due attention to the relevance of HM to ME (Grattan-Guiness 2004a, b). In the context of the *HPM perspective*, this is a distinction close to similar ones between pairs of methodological approaches; *explicit* & *implicit* use of history, *direct* & *indirect* genetic approach, *forward* & *backward* heuristics (Tzanakis, Arcavi *et al* 2000, pp.209-210). Hence, this distinction is potentially of great relevance to ME (Rogers 2009, 2011; Tzanakis & Thomaidis 2012), serving, among other things, to contribute towards answering the recurrent question "*Why and which history is appropriate to be used for educational purposes?*" (Barbin 1997).

2.3.2 *Which role can History of Mathematics play in Mathematics Education?*

Perhaps, this is the question that has been discussed and analyzed most on the basis of both a priori theoretical and epistemological arguments and of empirical educational research. At least implicitly, such analyses try to refute some of the above objections, especially those concerning the barriers posed by the complexity of the historical development (A(a2), A(a3), A(b1)) and/or by students' predisposition to and general knowledge of both mathematics and history as taught subjects (objections (B(b1), B(b2) and B(b3), A(b2), respectively).

This question has been extensively discussed from several points of view, especially in

relation to the appropriateness and pertinence of original historical sources in ME. In this context, HM can play three mutually complementary and supplementary roles (Barbin 1997; Jahnke *et al* 2000, §9.1; Furinghetti *et al* 2006, pp.1286-1287; Furinghetti 2012, §5; Jankvist 2013, §7):

A *replacement* role: Replacing mathematics as usually understood (a corpus of knowledge consisting of final results/finished and polished intellectual products; a set of techniques for solving problems given from outside; school units useful for exams etc.) by something different (to emphasize not only final results, but also mental processes that may lead to them; hence to perceive mathematics not only as a collection of well-defined and deductively organized results, but also as a vivid intellectual activity).

A *reorientation* role: Changing what is (supposed to be) familiar, to something unfamiliar; thus challenging the learner's and teacher's conventional perception of mathematical knowledge as something that has always been existing in the form we know it, into the deeper awareness that mathematical knowledge was an invention, an evolving human intellectual activity.

A *cultural* role: Making possible to appreciate that the development of mathematics takes place in a specific scientific, technological or societal context at a given time and place; thus becoming aware of the place of mathematical knowledge as an integral part of human intellectual history in the development of society; hence, seeing mathematics from perspectives that lie outside its nowadays established boundaries as a discipline.

Considered from the point of view of the *objective* of integrating HM in ME, there are 5 main areas in which the *HPM perspective* could be valuable:
The learning of mathematics;
The development of views on the nature of mathematics and mathematical activity;
The didactical background of teachers and their pedagogical repertoire;
The affective predisposition towards mathematics; and
The appreciation of mathematics as a cultural-human endeavor.

These are analyzed in detail into more specific arguments in Tzanakis, Arcavi *et al*, 2000, §7.2, describing in this way the role of history in the educational process.

From the point of view of the *way* HM is accommodated into this perspective, a distinction was made by Jankvist (2009); namely, history serving as a *tool* for assisting the actual learning and teaching of mathematics, and history serving as a *goal* in itself for the teaching and learning of the historical development of mathematics (see also Jankvist & Kjeldsen 2011). A similar distinction between *history for reflecting on the nature of mathematics as a socio-cultural process* and *history for constructing mathematical objects* was made by Furinghetti (2004; 2012, §5).

In this way, a finer and more insightful categorization of the possible roles of HM in ME resulted, reflecting the variety of their possible implementations in practice.

A small selection appears below (many more in section 3).

- Fauvel & van Maanen 2000: chs.7, 8 provide a variety of examples of possible classroom implementations, for several mathematical subjects; ch.9 gives examples of using original sources in the classroom and specific didactical strategies to do so.

- Katz & Tzanakis 2011, chs. 9, 10, 13, 14, 16, 19, and Sriraman 2012, chs. 2, 7, 14 provide particular examples, most of them emphasizing empirical results of actual implementations.

- Katz *et al* 2014: Rich on recent work in the *HPM domain*, including a sufficiently comprehensive old and recent bibliography in the editors' introduction and in its 12 papers. They concern theoretical issues on the history, philosophy and epistemology of mathematics,

and on empirical investigations both in school and teacher education.

- Doctoral dissertations with considerable work on both the theoretical issues of the *HPM perspective* and on empirical investigation and evaluation of actual implementations: e.g. van Amerom 2002; Su 2005; Clark 2006; Jankvist 2009a; Glaubitz 2010.

2.3.3 *To what extent the History of Mathematics has been integrated in Mathematics Education (curricula, textbooks, educational aids/resource material, teacher education)?*

Considerable work has been done over the last 15 years on understanding better and formulating more sharply the methodological issues raised by the integration of HM in ME, on producing appropriate educational aids of various types (B(a4)), and on designing and implementing teaching approaches to specific subjects and instructional levels in this context, with special emphasis on teacher education (B(a2), B(a3)).

According to the classification of the various approaches to integrate HM in teaching and learning mathematics given in Tzanakis, Arcavi *et al* 2000, there are three broad ways that may be combined (thus complementing each other), each one emphasizing a different aim:
To provide direct historical information, aiming to learn history;
To implement a teaching approach inspired by history (explicitly or implicitly), aiming to learn mathematics;
To focus on mathematics as a discipline and the cultural and social context in which it has been evolving, aiming to develop a deeper awareness of its evolutionary character, its epistemological characteristics, its relation to other disciplines and the influence exerted by factors both intrinsic and extrinsic to it.

From a methodological point of view, Jankvist 2009b classified the teaching & learning approaches in three categories:
Illumination approaches, in which teaching and learning is supplemented by historical information;
Module approaches, in the form of instructional units devoted to history, often based on specific cases;
History-based approaches, in which history shapes the sequence and the way of presentation, often without history appearing explicitly, but rather being integrated into teaching.

Approaches may vary in size and scope, according to the specific didactical aim, the mathematical subject matter, the level and orientation of the learners (A(b1), B(b3)), the available didactical time (B(a1)), and external constraints (curriculum regulations, number of learners in a classroom etc.).

The crucial role of teachers' training for effectively following the *HPM perspective* has been stressed repeatedly (e.g. Barbin *et al* 2000, p.70; Furinghetti 2004, p.4; Barbin *et al* 2011b; Alpaslan *et al* 2014, pp.160-162). Similarly, the need for appropriate didactical resources is equally crucial (e.g. Tzanakis, Arcavi *et al* 2000, pp.212-213; Pengelley 2011, pp.3-4).

Though accommodating the *HPM perspective* in an essential way into the official national curricula does not seem to have attained wide applicability,[3] intensive efforts have been made to train teachers and explore changes in their attitude and/or teaching, and to design, produce and make available didactically appropriate resources. Some indicative examples (more in section 3):

Teacher training: Bruckheimer & Arcavi 2000; Liu 2003; Waldegg 2004; Arcavi & Isoda

[3] One exception is Denmark (see Jankvist 2013, §3; Kjeldsen 2011b, §15.2; Niss & Højgaard 2011, ch.4). For a recent discussion and survey see Boyé *et al* 2011.

2007; Clark 2011; Smestad 2011; Mosvold *et al* 2014; Povey 2014.

***Resource material & educational aids*:** The need for didactical resources along the lines of the *HPM perspective* has been satisfied to a considerable extent in the last 15 years, so that such material is available nowadays in a variety of forms. Some examples:

- A wide spectrum of resource material can be found in *Convergence*; e.g. (i) *HPM Newsletter*, No90/2015, pp.10-12 for a recent sample; (ii) Clark 2009 (detailed description of a teaching module).
- Katz & Michalowicz 2005: didactical source material in 11 mathematical modules.
- Siu 2007: a useful survey of the literature and available resources.
- Pengelley *et al* 2009: Didactical material for discrete mathematics based on original sources.
- Pengelley & Laubenbacher 2014: A website with many references to published work and material available online.
- Barnett *et al* (2014): Extensive information on teaching with historical sources and bibliography on its theoretical framework and available resource material.
- Books with material that can be used directly and/or inspire teaching; e.g. Stein 2010 (review in *HPM Newsletter* No77/2011, pp.8-9); Barbin 2015 (review in *HPM Newsletter* No 89/2015, pp.13-14).

2.3.4 *How can this role be evaluated and assessed and to what extent it contributes to amend the teaching and learning of mathematics?*

Evaluating the effectiveness of the *HPM perspective* on improving ME from the point of view of both teaching and learning mathematics is an issue clearly stressed in objections B(c1), B(c2). Those who oppose, or are reserved about the role of HM in ME rightly ask for sufficient empirical evidence about its effectiveness. Quite early it has become clear that this is a key issue (e.g. Siu & Tzanakis 2004, p.3; Jankvist 2007), and that any such evaluation is a complex process relying more on qualitative, than quantitative methodologies: to consider changes induced in teachers' own perception of mathematics; to examine how this may influence the way they teach mathematics; and to explore if and in which ways this affects students' perception and understanding of mathematics (Barbin *et al* 2000, particularly §§3.1, 3.2).

Additionally, any such evaluation goes together with actual classroom implementations, in school teaching and teacher pre- and in-service education. Therefore, many, if not all, works referring to such implementations necessarily address evaluation issues about the effectiveness of the approach considered in each case (e.g. those listed in §§2.3.2, 2.3.3).

This is an area of currently active research with no established results of universal acceptance because of several reasons:

(a) Such a complex process is not expected to lead to spectacular changes in a short time interval. Preconceptions, misconceptions, predispositions either of the teachers or the students are too stable to be easily and/or quickly modified. Therefore, one should expect to see such changes after a considerable time exposure to an approach adopting the *HPM perspective*; often this time is not available.

(b) There is strong dependence on the instructional level (primary, secondary, tertiary) and orientation of the students, teacher-students included (science or humanities; elementary or secondary school teachers etc.), as well as, on their entire previous educational path, which has determined their knowledge of, attitude towards, and preconceptions about mathematics.

(c) There is influence by external "technical" factors that may favor, impede, or even prevent the implementation of an approach based on the *HPM perspective*: the curriculum and

the corresponding regulations; the number of students in the class (e.g. a small number facilitates group work and teacher's effective supervision); the structure of the educational system (e.g. in a centralized system, teachers have less freedom, hence fewer possibilities to apply an innovative teaching approach not necessarily falling into the official curriculum regulations).

(d) Not all mathematical subjects are equally accessible or appropriate to be taught and/or learned in a historically motivated/driven context.

All this constitutes a complex network of factors interfering with each other, so that empirical findings of different research works are not easily comparable. Therefore, despite many thoughtfully designed and carefully applied empirical investigations, much work is still needed to evaluate the effectiveness of the role of HM in ME in an undisputable way.

3. A bibliographical survey in the *HPM domain* since 2000

This section provides a comprehensive bibliographical survey of work related to the *HPM perspective* since 2000, indicating the TSG 25 themes to which each item is related and relevant, except those collective works that practically touch upon all themes (cf. remark (a) and Appendix in §1.2). Within each subsection, items appear by publication year and for each year by authors' alphabetical order. For those works included in Section 5 only author names and a note "see section 5" are given.

Throughout this study, journals, proceedings and conferences are abbreviated as follows:

ABBREVIATIONS

Journals

BSHM: BSHM Bulletin: Journal of the British Society for the History of Mathematics
ESM: Educational Studies in Mathematics
FLM: For the Learning of Mathematics
Int. J. ME Sci. Tech.: International Journal of Mathematical Education in Science and Technology
Int. J. Math. Teach. Learn: International Journal for Mathematics Teaching and Learning
Int. J. Sci. ME: International Journal of Science and Mathematics Education
JMB: Journal of Mathematical Behavior
JRME: Journal for Research in Mathematics Education
MER: Mathematics Education Review
MT: The Mathematics Teacher
MTL: Mathematical Thinking and Learning
MedJRME: Mediterranean Journal for Research in Mathematics Education
NOMAD: Nordic Studies in Mathematics Education
PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies
R-IREM: Repères-IREM
RME: Research in Mathematics Education
S&E: Science & Education
TJME: Taiwan Journal of Mathematics Education
TME: The Mathematics Educator
ZDM: ZDM - The International Journal of Mathematics Education

In Chinese:
MB: Mathematical Bulletin 数学通报

JME: Journal of Mathematics Education 数学教育学报

CTMM: Curriculum, Teaching Material and Method 课程·教材·教法

MTMS: Mathematics Teaching in Middle Schools (Junior High School)/ (Senior High School) 中学数学教学参考

ERR: Educational Researches and Reviews (High School Education and Instruction) 教育研究与评论

HSMM: The High School Mathematics Monthly 中学数学月刊

CER: Comparative Education Review 比较教育研究

Conferences

CERME: Congress of European Research in Mathematics Education

ESU: European Summer University on the History and Epistemology in Mathematics Education.

HPM XXXX: The HPM Group Satellite meeting of ICME-Y[4]

ICME: International Congress on Mathematical Education

ICTM: International Conference on the Teaching of Mathematics (at the undergraduate level)

Books

Hand Int RME: Handbook of International Research in Mathematics Education

ICMI Study Volume: J. Fauvel & J. van Maanen (Eds.), *History in Mathematics Educatio*n, *The ICMI Study*. Dordrecht: Kluwer, 2000.

Int. Hand. Res. Hi. Phil. Sci. Teach.: International Handbook of Research in History, Philosophy and Science Teaching

TMMEM: The Montana Mathematics Enthusiast Monographs

Other

ERME: European Society for Research in Mathematics Education

HPM Group: International Study Group on the Relations between the History and the Pedagogy of Mathematics

ICMI: International Commission on Mathematical Instruction

MAA: The Mathematical Association of America

NCTM: National Council of Teachers of Mathematics (USA)

Proc.: Proceedings

TSG: Topic Study Group

WG: Working Group

3.1 Collective works

3.1.1 *Collective volumes* in this area, with research papers, reviews of work etc.

2000

Dorier, J.-L. (Ed) *On the teaching of Linear Algebra*. Dordrecht: Kluwer. T2, 5

This volume consists of two parts: On an epistemological analysis of the historical development of vector space theory; and on didactical issues addressed and actual implementations at the undergraduate level, to which this analysis is related.

Fauvel & van Maanen (see section 5).

[4] XXXX being the year that ICME-Y took place.

Vol. 6 of the *New ICMI Study Series*: The outcome of a 4-year collectively realized international study of 62 scholars in 11 groups, each one authoring one chapter under the coordination of a convenor. A landmark in establishing the HPM domain.

Katz (see section 5).

26 chapters based on papers presented in the 1996 HPM Meeting: 7 on general issues of HM in ME; 5 on the teaching of a particular subject using the HM; 3 on teacher education; 11 on the HM.

2001

Barbin, É., Duval, R., Giorgiutti, I., Houdebine, J. & Laborde C. (Eds) *Produire et lire des textes de démonstrations*). Paris: Ellipses.

It addresses all problems encountered in learning proof, by following a variety of approaches: mathematical, historical, epistemological, didactical, linguistic, cognitive. Revealing the diversity of the points of view on proof and its teaching is the strong point of this book.

2002

Boniface, J. (Ed) *Les constructions des nombres réels dans le mouvement de l'arithmétisation de l'analyse. Comprendre les mathématiques par les textes historiques*. Paris: Ellipses.

Cerquetti, F. & Rodriguez A. (Eds) *Faire des mathématiques avec des images et des manuscrits historiques*. Créteil: CRDP de Créteil. T2a, 4

An empirical study with high school students on rediscovering geometry using various educational aids; among others, excerpts from original sources (16-18th centuries)

2003

Bekken & Mosvold (see section 5). T1, 4, 2a, 5

A continuation of Swetz *et al* 1995. Of its 27 chapters, 9 are related to the HPM perspective, the others concern either the HM or the history of ME.

2005

Shell-Gellasch, A. & Jardine, D. (Eds) *From calculus to computers: using the last 200 years of mathematics history in the classroom*. MAA notes 68. Washington DC: MAA. T2, 1, 5

22 contributions focusing on 19th and 20th century mathematics, emphasizing recent history in the teaching of mathematics, computer science, and related disciplines; for details see http://www.maa.org/publications/books/from-calculus-to-computers (accessed 18/2/2016)

2006

Thomaidis, Y., Kastanis, N. & Tzanakis, C. (Eds) *History and Mathematics Education*. Thessaloniki: Ziti Publications (in Greek). T4, 5, 1

Collective volume with 5 papers on the HPM perspective; 6 on the history of ME and 3 on historiographic approaches to ancient Greek mathematics (more in *HPM Newsletter*, No63/2006, pp.7-9).

2007

Barbin, E. & Bénard, D. (Eds) *Histoire et Enseignement des Mathématiques: Rigueurs, erreurs, raisonnements*. Lyon: Institut National de Recherche Pédagogique. T5, 4, 1

12 chapters on rigor, experiment and proof in geometry, reasoning between geometry and algebra, and multiplicity of points of view in analysis. Contributions are based on original texts and anchor on epistemological and didactical reflections.

Boero, P. (Ed) *Theorems in School. From History, Epistemology and Cognition to Classroom Practice*. Rotterdam: Sense Publishers. T5, 1

15 chapters in 4 sections exploring: the historical-epistemological nature of proof; the way national mathematics

curricula and the underlying epistemological beliefs might influence students' perceptions of proof; the similarities and differences between argumentation and proof; the teaching of proof.

2008

Shell-Gellasch, A. (Ed) *Hands on History: A Resource for Teaching Mathematics*. MAA notes 72. Washington DC: MAA. T2b, 5, 6

16 chapters on historical examples appropriate for incorporating hands-on learning: From simple devices, to elaborate models of descriptive geometry, and detailed descriptions on how to build and use historical models in the high school or collegiate classroom; for details see http://www.maa.org/press/books/hands-on-history-a-resource-for-teaching-mathematics (accessed 18/2/2016)

Robson, E. & Stedall, J. (Eds) *The Oxford Handbook of the History of Mathematics.* Oxford: Oxford University Press. T6, 5

This is not just a 'history of mathematics'; it is unusual in that it is organized in 3 major themes: geographies and cultures; people and practices; interactions and interpretations that are sensitive to modern historiography and include social and cultural backgrounds and epistemologies.

2009

Greek Society of the Didactics of Mathematics (Eds). *The value of the history of mathematics in mathematics education*, Thessaloniki: Ziti Publications (in Greek). T2, 4, 1

9 contributions on one or more of the following issues: Whether and how crucial historical steps can be integrated into ME; whether and to what extent historical knowledge allows the prediction of students' difficulties; whether the integration of HM in ME is possible at all, given the constraints imposed by the official curriculum and the teachers' and students' established preconceptions.

2010

Barbin, E. (Ed) *Des grands défis mathématiques d'Euclide à Condorcet*. Paris: Vuibert. T2, 4, 5, 1

9 examples of introducing a historical perspective in ME, having as a starting point specific historical problems and organized in 4 parts: measuring magnitudes, representing magnitudes, calculating the probable, approaching a curve; thus illustrating the great domains of today's taught math: analysis, algebra, probability and geometry (for details see http://culturemath.ens.fr/node/2582 (accessed 18/2/2016) or *HPM Newsletter* No75/2010, pp.1-2).

Hanna, G., Jahnke, N. & Pulte, H. (Eds) *Explanation and proof in mathematics: philosophical and educational perspectives*. New York: Springer. T1, 5

The 17 chapters assemble perspectives from ME, its history and philosophy to strengthen mutual awareness and share recent findings and advances in these interrelated fields. By a variety of examples, the authors explore the role of refutation in generating proofs, the varied links between experiment and deduction, the use of diagrammatic thinking in addition to pure logic, and the uses of proof in ME.

2011

Katz & Tzanakis (see section 5).

An all-embracing outcome of activities within the HPM Group during 2007-2009; to present an overview of the state of the art in this area after the appearance of the ICMI Study volume. 7 chapters on theoretical aspects of HM in ME; 10 with concrete implementations; 4 with particular focus on teacher education; 3 invited papers on the HM (indicative examples in section 5).

Shell-Gellasch, A. & Jardine, D. (Eds) *Mathematical Time Capsules: Historical Modules for the Mathematics Classroom*. MAA notes 77. Washington DC: MAA. T2b, 5

35 chapters offering teachers historical modules for immediate use in the math classroom (undergraduate or secondary math curricula). Each capsule presents at least one topic or a historical thread that can be used

throughout a course and further references and resources on the chapter subject; for detail see http://www.maa.org/press/ebooks/mathematical-time-capsules (accessed 18/2/2016).

2012

Barbin, E. (Ed) *Les mathématiques éclairées par l'histoire: des arpenteurs aux ingénieurs*. Paris: Vuibert. T2, 4, 6

The 9 chapters provide examples of the influence or use of HM in the math classroom. Authors work with students from upper secondary school to tertiary, including teacher training. They show how their own reading and reflection has led to direct use of historical material in the classroom, either through the use of original texts or by devising tasks based on the methods or examples of their subjects (*HPM Newsletter* No80/2012, pp.12-13).

Chemla, K. (Ed) *The History of Mathematical Proof in Ancient Traditions*. Cambridge University Press. Cambridge, UK. T2b, 6

A number of important contributions from international scholars on the historiography, history and epistemology of mathematical proof, relevant for today's mathematics classroom.

Sriraman, B. (Ed.). *Crossroads in the History of Mathematics and Mathematics Education*, TMMEM vol.12. Charlotte NC: Information Age Publishing. T1, 2b, 5

15 chapters: 7 in the history and didactics of calculus and analysis, 4 in the history and didactics of geometry and number and 4 on HM in ME.

2013

Barbin, E. & Moyon, M. (Eds) *Les ouvrages de mathématiques dans l'histoire. Entre recherche, enseignement et culture*. Limoges: PULIM. T3b

For contents & summary, see http://www.pulim.unilim.fr/index.php/notre-catalogue/fiche-detaillee?task=view&id=750 (accessed 18/2/2016).

2014

Matthews, M. (Ed) *Int. Hand. Res. Hi. Phil. Sci. Teach*. Dordrecht: Springer. T1, 4, 5, 3b

Of the 76 chapters, 7 are devoted to ME, 4 of which concern the HPM perspective.

2015

Barbin, É. & Cléro, J.-P. (Eds) *Les mathématiques et l'expérience: ce qu'en ont dit les philosophes et les mathématiciens*. Paris: Hermann. T5, 6

17 contributions on philosophical issues, pertinent to mathematics and its epistemological characteristics, on the basis of texts of various philosophers through the ages; for details see http://les-livres-de-philosophie.blogspot.gr/2015/10/barbin-evelyne-clero-jean-pierre-dir.html (accessed 18/2/2016).

Barbin, É. (Ed) *Les Constructions Mathématiques avec des Instruments et de Gestes (Mathematical Constructions: making and doing)*. Paris: Ellipses-Editions. T2b

Any study of geometry involves both thinking and doing and it is through the doing (drawing, measuring, copying) that we develop a sense of what geometry is. School mathematics has today lost much of its geometry. It is in an attempt to recall this loss, and to report on the history of geometrical constructions, that provides the spur for this collection (see *HPM Newsletter* No 89/2015, pp.13-14).

Rowe, D.E., & Horng, W.S. (Eds) *A Delicate Balance: Global Perspectives on Innovation and Tradition in the History of Mathematics: A Festschrift in Honor of Joseph W. Dauben*. Berlin: Birkhäuser. T6

A collection of contributions on various historical issues, from the perspective and/or in the context of different cultures; for content & summary see http://www.springer.com/gb/book/9783319120294 (accessed 18/2/2016).

To appear

Jardine, D., & Shell-Gellasch, A. (Eds) *The Courses of History: Ideas for Developing History*

of Mathematics Courses, Washington, DC: MAA. T1, 3b, 4, 5, 6

3.1.2 *Special issues* of international journals of ME

2004

Siu & Tzanakis (see section 5). T1, 2, 3a, 5

A special double issue with 10 papers originally presented at ICME 10, TSG 17 "The role of the History of Mathematics in Mathematics Education" on epistemological issues; teacher education; didactical material; particular examples.

2007

Furinghetti, F., Radford, L. & Katz, V. (Eds) The History of Mathematics in Mathematics Education: Theory and Practice. *ESM* **66**(2), 107-271. T1, 2a, 3b, 4

A special issue with 10 papers, seeking to deepen the understanding of the pedagogical role HM may play in contemporary ME. Some provide examples of the use of the HM in school practice and teacher education; others address theoretical questions that have become crucial to understanding the profound intertwining of past and present, conceptual developments on spreading new epistemologies and theories of learning.

2010

Stedall, J. (Ed) Special Issue: The history of mathematics in the classroom. *BSHM* **25**(3), 131-179. T2a

4 articles on the HM in classroom practice, from teaching basic arithmetic in primary school to teaching statistics to young adults.

2014

Clark, K. & Thoo, J.B. (Eds) The Use of History of Mathematics to Enhance Undergraduate Mathematics Instruction. *PRIMUS* **24**(8), 663-773. T2, 4, 6

8 papers focused on undergraduate math teaching and learning using HM as a vehicle: 4 on some aspect of including HM within coursework required of undergraduate students; 4 on authors' experiences with developing and using primary source material with undergraduate math students. Results from: classroom experiments; teaching HM courses for math, ME, or philosophy majors; ways of integrating original sources in the undergraduate classroom.

Katz *et al* (see section 5). T1, 2a, 5

Special issue with an introduction accompanied by an extensive bibliography and 12 papers directly related to the HPM perspective, divided into 4 sections: theoretical issues in the use of the HM in teaching; direct uses of the HM in the classroom; HM in teacher education; relations between the philosophy, the epistemology, the teaching and the sociology of mathematics.

Kourkoulos, M. & Tzanakis, C. (Eds) History of Mathematics and Mathematics Education. *Education Sciences*. Special Issue for 2014, 5-198. T1, 2a, 3a

Bilingual issue with 9 papers related to the *HPM perspective* (3 in English, 6 in Greek): 3 concern general ideas, conceptual frameworks and methodological schemes and 5 refer to specific issues with focus on classroom implementations (from elementary school to the university).

2015

Karam, R. Thematic issue: The Interplay of Physics and Mathematics: Historical, Philosophical and Pedagogical Considerations, *S&E* **24** (5-6), 487-805. T1, 5, 6, 2a

11 papers challenging the typical situation found in educational contexts: In physics education, math is a mere tool to describe and calculate; in ME, physics is only a possible context for applying mathematical concepts previously defined abstractly. Overcoming this dichotomy (which creates significant learning problems for the students)

demands a systematic research effort in different fields, especially when aiming at informing educational practices by reflecting on historical, philosophical and sociological aspects of scientific knowledge.

2016

Nikolantonakis, K.: The use of History in Mathematics Education, *Menon*, Thematic Issue **2**, 4-158. T2a, T4, T5

Thematic bilingual issue with 8 papers related to the *HPM perspective* (6 in English, 2 in French): Most of them refer to/are based on empirical studies (4 on elementary and/or junior high school and 4 on pre- and in-service teacher education) and are related to the use of original sources, either directly in the classroom, or/and as a source of inspiration and rethinking on basic issues in ME.

3.1.3 *Proceedings of conferences and meetings* (with reference to their accessibility via the Internet, wherever possible)

2000

Horng & Lin (see section 5).

5 plenary lectures, 5 round tables, 4 panels, 35oral presentations, 1 workshop with special emphasis on the effectiveness of the HPM perspective, cultural aspects and history of ME.

2004

Horng *et al* (see section 5).

Proceedings of a meeting preceding HPM 2004; see HPM Newsletter No55/2004, pp.12-14; No56/2004, p.16.

2006

Furinghetti *et al* (see section 5). T4, 1, 2a

Workshop on the original sources in ME. Research questions were identified which evolved from work in the past and are helpful in orienting future work. They reflect central issues related to the integration of original sources from the HM into ME, both in learning and teaching mathematics (see *HPM Newsletter*, No62/2006, pp.7-10).

Furinghetti, F., Kaisjer, S. & Tzanakis, C. (Eds) *Proc. of HPM 2004 & ESU 4*, Iraklion: University of Crete, Greece (accessed 18/2/2016).

Revised edition of the *HPM 2004 of ICME 10* & the *4th ESU* proceedings (Furinghetti *et al* 2004). 78 papers divided into 6 sections, corresponding to the 6 main themes of this joint meeting.

2008

Barbin *et al* (see section 5).

78 full papers & 42 abstracts, in 6 sections corresponding to the ESU 5 six main themes: 6 plenary lectures, 2 panel discussions, 19 workshops based on didactical & pedagogical material, 25 workshops based on historical & epistemological material, 44 oral presentations and 26 short communications.

Cantoral *et al* (see section 5).

70 papers on the 6 themes of the meeting, stemming from 4 plenary lectures, 5 workshops and 61 oral presentations.

2010

Furinghetti, F., Dorier, J.-L., Jankvist, U.T., van Maanen, J. & Tzanakis, C. (Eds) WG 15: Theory and research on the role of history in mathematics education. In V. Durand-Guerrier, S. Soury-Lavergne, F. Arzarello (Eds), *Proc. of CERME 6* (pp.2677-2810). Lyon: Institut National De Recherche Pédagogique. T2a, 4, 3b, 5 (accessed 18/2/16)

Group work structured along 8 main themes, leading to 14 papers included in this proceedings.

2011

Barbin *et al* 2011a (see section 5).

55 texts and 35 abstracts divided into 6 sections corresponding to the ESU 6 six main themes (see *HPM Newsletter* No79/2012, pp.9-11).

Jankvist, U.T., Lawrence, S., Tzanakis, C. & van Maanen, J. (Eds) WG 12: History in Mathematics Education. In M. Pytlet, T. Rowland, E. Swoboda (Eds), *Proc. of CERME 7* (pp.1636-1779). Rzeszow, Poland: University of Rzeszow. T1, 2a, 4 (accessed 18/2/2016).

Group work structured along 4 general topics, leading to 15 papers included in this proceedings. Topics: Research questions and relevance of research; use of HPM theory and ME theory; methods, data and analysis; validity, reliability and generality of research results.

2012

Barbin *et al* (sec. 5).

78 contributions on the 7 themes of the meeting, stemming from 7 plenary lectures, 7 workshops, 57 oral presentations, 5 posters & 2 exhibitions. Revised version in progress.

2013

Jankvist, U.T., Clark, K., Lawrence, S. & van Maanen, J. (Eds) WG 12: History in Mathematics Education. In B. Ubuz, Ç., Haser, & M.A. Mariotti (Eds) *Proc. of CERME 8* (pp.1945-2067). Ankara: Middle East Technical University. T2a, 4, 5, 6 (accessed 18/2/2016).

Group work structured along 5 general themes leading to 13 papers and 3 posters included in this proceedings. Themes: interdisciplinarity; theoretical frameworks in history of ME; history in pre high school ME; history in high school ME; HM in teacher education and design.

2015

Barbin *et al* (see section 5). T1, 2a, 2b, 4, 6

53 texts and 34 abstracts corresponding to all types of activities during ESU 7, divided into 8 sections corresponding to its 7 themes and a poster session.

Chorlay, R., Jankvist, U.T., Clark, K., & van Maanen, J. (Eds.) WG 12: History in Mathematics Education. In K. Krainer & N. Vondrová (Eds) *Proc. of CERME 9* (pp.1778-1884). Prague: Charles University in Prague, Faculty of Education & ERME (accessed 4/4/2016).

Group work structured along 9 themes leading to 14 papers and 2 posters included in this proceedings, dealing with four areas of questions on history in ME-the student perspective; history in ME-the teacher perspective; history of ME-the mathematical education landscape; methodological reflections on history in/of ME.

3.1.4 *Resource material*, collectively produced, especially material available on the web.

2002

Bazin, M., Tamez, M. & Exploratorium Teacher Institute. *Mathematics and Science across Cultures. Activities and investigations from the Exploratorium*. New York: The New Press. T2b, 6

Addressed to grades 4-12 teachers, wishing to develop hands-on activities in classroom suggested by artefacts or instruments produced by different current and past civilisations.

2004

Barnett, J., Bezhanishvili, G., Leung, H., Lodder, J., Pengelley, D. & Ranjan, D. *Teaching discrete mathematics via primary historical sources* (accessed 18/2/2016). T2a, 2b, 4

Resource material on teaching projects via original sources, its rationale and extensive bibliography.

2005

IREM Basse-Normandie. *L'espérance du Hollandais ou le premier traité du calcul du hasard.* Paris: Ellipses. T2b

Annotated presentation of Huygens' book "De rationiciis in ludo aleae" (the 1st treatise on probability calculus 1657) and the commentaries on it by its contemporaries and successors.

Katz & Michalowicz (see section 5); T2b

An electronically available resource based on work done by a larger group of people, with material on 11 mathematical modules (secondary education), useful in the classroom and/or for teachers to adapt it according to their needs and those of their classroom

2006

Demattè, A. & Furinghetti, F. *Fare matematica con i documenti storici: Una raccolta per la scuola secondaria di primo e secondo grado*. Trento: Editore Provincia Autonoma di Trento – IPRASE del Trentino. T2b

Includes selection of excerpts from original sources and aims to provide secondary school teachers with activities for their integration in the classroom; thus promoting alternative ways of teaching through text-based activities and exercises in order to consolidate or/and introduce mathematical skills (one book for the teacher and one for the student; see *HPM Newsletter* No64/2007, pp.6-8).

Groupe d'Histoire & d'Epistémologie des Mathématiques de l'IREM de Lyon. *Textes fondateurs du calcul infinitésimal.* Paris: Ellipses. T2b

Annotated texts of Newton and Leibniz on infinitesimal calculus ; for details see http://www.decitre.fr/livres/si-le-nombre-m-etait-conte-9782729868222.html (accessed 18/2/2016).

Swetz, F.J. *Shu Xue You Ji*, (*The History of Mathematics: A Study Guide*) Taipei: Transoxania Ltd (in Chinese). T2b

Based on an earlier teaching course on HM.

2007

Katz, V. (Ed) *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook*. Princeton: Princeton University Press. T2b, 6.

English translations of key mathematical texts from the 5 most important ancient and medieval non-Western mathematical cultures, putting them into full historical and mathematical context. A firsthand understanding and appreciation of these cultures' important contributions to world mathematics. An essential resource and indispensable guide for math teachers who want to use non-western mathematical ideas in the classroom.

Siu, M.K. *Some Useful References for Course MATH2001 (Development of Mathematical Ideas) Department of Mathematics, University of Hong Kong* (accessed 18/2/201). T2b

An extensive list of papers, collective volumes, individual books, special journal issues, websites on the HM and its relations to ME. References either in English or Chinese. A useful resource for those interested in the HPM perspective.

2009

Clark (see section 5). T2b, 4

Introduces undergraduate students preparing to teach mathematics to Stevin's pamphlet on decimal fractions and to encourage prospective math teachers to think about connections with pupils' initial learning of decimals' multiplication.

Pengelley *et al* (see section 5). T2b, 4

An extended presentation focusing on the pedagogy of historical projects, which offer excerpts from original sources, place the material in context, and provide direction to the subject matter.

Rogers, L. *Resources from Leo Rogers History and the Mathematics Curriculum* (accessed 18/2/2016). T2b

A website with a lot of information; useful websites; bibliography; suggestions/advices for HM in ME etc.

2012

Swetz, F.J. *Mathematical Expeditions: Exploring Word Problems across the Ages.* Baltimore, MD: Johns Hopkins University Press. T2b, 4, 6

Review at http://www.maa.org/press/periodicals/convergence/review-of-emmathematical-expeditions-exploring-word-problems-across-the-agesem (accessed 18/2/2016)

2013

Barnett, J., Bezhanishvili, G., Leung, H., Lodder, J., Pengelley, D., Pivkina, I., Ranjan, D. & Zack, M. Primary Historical Sources in the Classroom: Discrete Mathematics and Computer Science. *Convergence*, **10**. DOI:10.4169/loci003984 T2b

Presentation of 16 separate curricular modules, each a project for students based on excerpts from primary historical sources, to provide context, motivation and direction for selected topics in discrete mathematics and computer science as an alternative form of instruction.

Siu, M.K. A reading list for teachers interested in history of mathematics.
See http://hkumath.hku.hk/~mks/Whynot%20Histh_MKS_XZYS.pdf and http://hkumath.hku.hk/~mks/HistMathTeachingReadingListChinese.pdf (accessed 18/2/2016). T2b

An interesting list of books, collective volumes, and papers on the HM and on history in ME.

2014

Friedelmeyer, J-P. & Lubet, J-P. (Eds) *L'analyse algébrique – un épisode clé de l'histoire des mathématiques.* Paris: Ellipses. T2b

Original texts by Euler, Lagrange, Brisson, Servois and others, contextualized by appropriate comments and introductions.

Pengelley, D. & Laubenbacher, R. (see section 5). T2a, 2b, 4

A website with information and materials on using original historical sources in math teaching, including the authors' own experiences and materials, and those of others.

2015

Commission interIREM, Commission IREM de Basse-Normandie, Barbin, É. & Legoff, J-P. *Si le nombre m'était compté*. Paris: Ellipses. T2b

A useful resource for the generalization of the number concept beyond natural numbers

3.2 Individual books and papers

3.2.1 *Books and Doctoral Dissertations*

2001

Marshall, G.L. *Using history of mathematics to improve secondary students' attitudes towards mathematics*. Dissertation. Illinois State University, Bloomington−Normal, Illinois. T2a

2002

Pizzamiglio, P. *Matematica e Storia. Per una didattica interdisciplinare*. Brescia: Ed. La Scuola. T5

van Amerom, B.A. (see section 5). T2, T1, T3a

2003

Scimone, A. *Pupils' conceptions about a historical open question: Goldbach's conjecture. The improvement of mathematical education from a historical viewpoint*. Dissertation. University of Bratislava, Bratislava. T1, 2a

2004

da Silva Souza, E. A prática social do cálculo escrito na formação de professores: A história como possibilidade de pensar questões do presente. Dissertation. UNICAMP, Campinas, Brazil. T2a

2005

Su (see section 5). T2a

van Gulik-Gulikers, I. *Een studie naar de waarde en de toepassing van de geschiedenis van de meetkunde in het wiskundeonderwijs*. Dissertation. Rijksuniversiteit Groningen, The Netherlands. T2a, 3a, 5

2006

Bagni, G.T. *Linguaggio, Storia e Didattica della Matematica*. Bologna: Pitagora Editrice.

Clark (see section 5). T2a, 3a

Thomaidis, Y., Petrakis, Y., Touloumis, K. & Stafylidou M. *Language, History and Euclidean Geometry*. Thessaloniki: University of Macedonia (in Greek). T2a, 2b, 4, 5

2007

Goodwin, D.M. *Exploring the relationship between high school teachers' mathematics history knowledge and their images of mathematics*. Dissertation. University of Massachussets, Lowell MA (accessed 18/2/2016). T3a, 2a

Knoebel, A., Laubenbacher, R., Lodder, J. & Pengelley, D. *Mathematical masterpieces – further chronicles by the explorers*. New York: Springer. T2b

2008

de Jesus Brito, A. *A Geometria de Euclides a Lobatschweski. Um esstudo histórico-pedagógico*. Natal, RN, Brazil: Editora da UFRN. T2b

Filloy, E., T. Rojano & L. Puig. *Educational Algebra. A Theoretical and Empirical Approach*. New York: Springer. T1, 5, 2a

Haile, T. *A Study on the Use of History in Middle School Mathematics: The Case of Connected Mathematics Curriculum*. Dissertation. Faculty of the Graduate School of the University of Texas, Austin (accessed 18/2/2016). T3a,b, T1

Stedall, J. *Mathematics Emerging: A Sourcebook 1540 – 1900*. Oxford: Oxford UP. T2b, 4

2009

Jankvist (2009a) (see section 5). T1, 2a

2010

Stein (see section 5). T2b

Wardhaugh, B. *How to Read Historical Mathematics*. New Jersey: Princeton University Press. T4

Glaubitz (see section 5). T2a, T4

2012

Ostermann, A. & Wanner, G. (2012). *Geometry by its History*. Berlin: Springer. T2b, T4, T5

2015

Shell-Gellasch, A. & Thoo, J. *Algebra in Context: Introductory Algebra from Origins to Applications*. Baltimore, MD: Johns Hopkins University Press. T2b

Guillemette, D. (2015). L'histoire des mathématiques et la formation des enseignants du secondaire: sur l'expérience du dépaysement épistémologique des étudiants. Dissertation. Montréal: Université du Québec à Montréal (accessed 15/6/2016). T1, T2a, T4, T5

3.2.2 *Individual papers in Scientific Journals*

2000

Arcavi, A., & Bruckheimer, M. Didactical uses of primary sources from the history of mathematics. *Themes in Education*, **1**(1), 55–74. T4, T2

Barbin, É. Que faut-il enseigner? Pour qui? Pourquoi? Des réponses dans l'histoire des mathématiques, in *R-IREM*, **38**, 43-51. T2b

Barbin, É. Construire la géométrie élémentaire. *R-IREM*, **40**, 5-9. T1, 5

Barbin, É. Pourquoi démontrer? Les mathématiques grecques, *Cahiers de Science & Vie*, **55**, 42-49. T1

Bkouche, R. Sur la notion de perspective historique dans l'enseignement d'une science. *R-IREM*, **39**, 35-59. T1

Furinghetti, F. The history of mathematics as a coupling link between secondary and university teaching. *Int. J. ME Sci. Tech.*, **31**(1), 43–51. T1, 2

Horng, W-S. Nam Byung Gil's comments on the "Tian Yuan Shu" versus "Jie Gen Fang": An HPM perspective. *Chinese Journal of Science Education*, **8**(3), 215-224 (in Chinese). T2b

Li, B.C. A survey on mathematics teachers' knowledge about the history of mathematics. *MB*, 39(3), 39-40. T3a

Matos, J. The historical development of the concept of angle (1). *TME*, **10**(2), 49-56. T2

Testa, G. L'enseignement des coniques à travers une approche historique: comment saisir un texte? *R-IREM*, **41**, 105-119. T4, T2

Tzanakis, C. & Thomaidis, Y. Integrating the close historical development of mathematics and physics in mathematics education: Some methodological end epistemological remarks. *FLM*, **20**(1), 44–55. T1, 5

2001

Barbin, É. Qu'est-ce que faire de la géométrie? *R-IREM*, **43**, 59-83. T1, T5, T4, T6

D'Ambrosio, U. What is ethnomathematics, and how can it help children in schools? *Teaching Children Mathematics*, **7**(6), 308., T6, T1

Filep, L. The development, and the developing of, the concept of a fraction, *International Journal for Mathematics Teaching and Learning*. Available from http://www.cimt.plymouth.ac.uk/journal/lffract.pdf (accessed 18/2/201). T2b

Fried (see section 5). T1

Friedelmeyer, J.-P. Grandeurs et nombres: l'histoire édifiante d'un couple fécond. *R-IREM*, **44**, 5-31. T2a

Furinghetti, F. & Somaglia, A. The method of analysis as a common thread in the history of

algebra: reflections for teaching. *Themes in Education*, **2**(1), 3-14. T2a, 4, 1

Jahnke, H.N. Cantor's cardinal and ordinal infinities: An epistemological and didactic view. *ESM*, **48**, 175–197. T1, 2

Lit, C-K., Siu, M.K. & Wong, N.Y. The use of history in the teaching of mathematics: Theory, practice, and evaluation of effectiveness. *Educational Journal*, **29**(1), 17–31. T2, 1

Mendez, E.P. A history of mathematics dialogue in textbooks and classrooms, *MT* **94**(3), 170-173. T2, 3b

Percival, I. An artefactual approach to ancient arithmetic. *FLM*, **21**(3), 16-21. T2a

Povey, H., Elliott, S. & Lingard, D. The study of the history of mathematics and the development of an inclusive mathematics: Connections explored. *MER*, **14**, 8–18. T2a, 1

van Gulik-Gulikers, I. & Blom, K. 'A historical angle', a survey of recent literature on the use and value of history in geometrical education. *ESM*, **47**(2), 223–258. T3

2002

Boyé, A. & Comairas, M.-C. Moyenne, médiane, écart-type, quelques regards sur l'histoire pour éclairer l'enseignement des statistiques au lycée. *R-IREM* **48**, 27-40. T2b, 5

Fleener, M., Reeder, S., Young, E. & Reylands, A. History of mathematics: Building relationships for teaching and learning. *Action in Teacher Education*, **24**, 73-84. T2a

Nooney, K.A. Critical question: Why can't mathematics education and history of mathematics coexist? *TME*, **12**(1), 1–5. T1

Quinton, P. Activités mathématiques à propos de la mesure de la Terre. *R-IREM*, **49**, 73-92. T2b, a, 5

2003

Friedelmeyer, J.-P. Euclide peut-il encore apprendre quelque chose au professeur de mathématiques d'aujourd'hui ? *R-IREM*, **53**, 23-42. T2b, 4

Furinghetti, F. & Paola, D. (2003). History as a crossroads of mathematical culture and educational needs in the classroom. *Mathematics in School*, **37**(1), 37-41. T2b,a, 1

Glaubitz, M. & Jahnke, H. N. Die Bestimmung des Umfangs der Erde als Thema einer mathematikhistorischen Unterrichtsreihe, *Journal für Mathematik-Didaktik*, **24**(2), 71-95. T4

Glaubitz, M. & Jahnke, H. N. Texte lesen und verstehen. Eine Erwiderung. *Journal für Mathematik-Didaktik*, **24** (3/4), 252-260. T4

Guichard, J.-P. D'un problème de Diophante aux identités remarquables. *R-IREM*, **53**, 5-19. T2b, 4, 6

Jahnke, H. N. Numeri absurdi infra nihil. Die negativen Zahlen. *Mathematik lehren*, **121**, Dez. 2003, 21-22; 36-40. T2b

Liu (see section 5). T1

Ransom, P. Drawing instruments – their history and classroom use. *BSHM*, **47**, 53-56 & **48**, 45-48. T2b

Streefland, L. Learning from history for teaching in the future. *ESM*, **54**, 37–62. T1, 5, 2a

Tournès, D. Du compas aux intégraphes: les instruments du calcul graphique. *R-IREM*, **50**, 63-84. T2b

van Amerom, B. A. Focusing on informal strategies when linking arithmetic to early algebra. *ESM*, **54**, 63–75. T2a, T4

van Gulik-Gulikers, I. The seventeenth-century surveyor in class. *BSHM*, **47**, 56-63. T2a, 4

Wang, X.Q., & Ouyang, Y. Historical notice on HPM. *JME*, 12(3): 24-27. T1

2004

Barbin, É. L'outil technique comme théorème en acte, *Sciences et avenir*, **140,** 26-27. T1

Grattan-Guiness 2004a (see section 5). T1

Grattan-Guiness 2004b (see section 5). T1

Keiser, J.M. Struggles with developing the concept of angle: Comparing sixth-grade students' discourse to the history of the angle. *MTL*, **6**(3), 285-307. T2a, T1

Li, H.T. The new horizon of the curriculum reform: The history of mathematics entering the new mathematics curriculum. *CTMM*, **2**5(9), 51-64. T1

Lopez-Real, F. Using the history of mathematics as a starting point for investigations: some examples on approximations. *Teaching Mathematics Applications*, **23**(3), 133-147. T2b

Mercier J.-P. Le problème des cinq carrés ou comment montrer l'intérêt des identités remarquables. *R-IREM*, **57**, 47-67. T2b, 4, 6

Merker, C. La méthode des indivisibles racontée lors d'un stage. *R-IREM*, **54**, 57-76. T2b, 4

Métin, F. Avez-vous lu Euclide, *R-IREM*, **55**,101-103. T2b

Michalowicz, K. & McGee, R. Using historical problems in the middle school, *Convergence*, **1** (accessed 18/2/2016). T2b, 4

Swetz, F.J. Using problems from the history of mathematics, *Convergence*, **1** (accessed 18/2/2016). T2b, T6, T4

2005

Burn, B. The vice: Some historically inspired and proof-generated steps to limits of sequences. *ESM*, **60**, 269–295. T2b

Carroget, L. & Gairin C. Les Mathématiques sans ignorer nos anciens. *R-IREM*, **58**, 5-15. T2a, 1

Dubinsky, E., Weller, K., McDonald, M.A. & Brown, A. Some historical issues and paradoxes regarding the concept of infinity: An APOS-based analysis: Part 1, *ESM*, **58**(3), 335-359. T1

Durand-Guerrier, V. & Arsac, G. An epistemological and didactic study of a specific calculus reasoning rule, *ESM*, **60**(2), 149 – 172. T2a, 5, 1

Su, Y.W. Designing HPM work sheets. *Taiwan Journal of Mathematics Teachers*, **2**, 2-13 (in Chinese). T2b

Wang, X.Q. The Historical-genetic principle viewed from a test on the imaginary numbers and divergent series. *JME*, **14**(3), 30-33. T3b

2006

Bakker A. & Gravemeijer, K P.E. An historical phenomenology of mean and median. *ESM*, **62**(2), 149 – 168. T1, 2a

Barbin, É. Apports de l'histoire des mathématiques et de l'histoire des sciences dans l'enseignement, *Tréma*, **26**, 20-28. T1, 5

Liu, P-H. & Niess, M.L. An exploratory study of college students' views of mathematical thinking in a historical approach calculus course. *MTL*, **8**(4), 373-432. T2a

Martínez-Torregrosa, J., López-Gay, R. & Gras-Martí, A. Mathematics in physics education: Scanning historical evolution of the differential to find a more appropriate model for teaching

differential calculus in physics. *S&E*, **15**(5), 447–462. T5

Savizi, B. Applicable problems in history of mathematics; practical examples for the classroom, *Philosophy of Mathematics Education Journal*, **19** (accessed 18/2/2016). Also in *Teaching Mathematics and its Applications*, **26** (2007), 51-54. T1, 2b

Thiénard, J.-C. Les transformations en géométrie, introduction à une approche historique. *R-IREM*, **63**, 27-52. T2b, 4

Tillema, E. Chinese algebra: Using historical problems to think about current curricula. *MT*, **99**(4), 238-245. T2, 6

Wang, X.Q. & Zhang, X.M. Researches on HPM: Contents, methods and examples. *JME*, **15**(1), 16-18. T1

2007

Barbin, É. L'arithmétisation des grandeurs. *R-IREM*, **68**, 5-20. T2b, 5

Barbin, É. Les Récréations: des mathématiques à la marge, *Pour la science*, **30** 22-25. T3

Barbin, É. Les avatars de la rigueur mathématique, *Pour la science*, **356**, 10-13. T1

Carson, R. N. & Rowlands, S. Teaching the Conceptual Revolutions in Geometry, *S&E*, **16** (9-10), 921-954. T2b, 1

Demattè, A. (2007). Primary sources from the history of mathematics for secondary school students. *Acta Didactica Universitatis Comenianae Mathematics*, **7**, 47-66. T2b, 4, 1

Farmaki V. & Paschos, Th. Employing genetic 'moments' in the history of mathematics in classroom activities. *ESM*, **66**(1), 83–106. T2a, 5

Gérini ,C., & Verdier, N. Les Annales de Gergonne (1810-1832) et le Journal de Liouville (1836-1874): une mine de textes numérisés à exploiter dans notre enseignement. *R-IREM*, **67**, 55-68. T2b

Jankvist (see section 5). T3a

Li, T. A. & Song, N. Q. Teaching of Analytic Geometry from the Historical Perspective. *JME*, **16**(2): 90-94. T1

Meavilla, V. & Flores, A. History of mathematics and problem solving: a teaching suggestion. *Int. J. ME Sci. Tech.*, **38**(2), 253-259. T2

Ren, M. J., & Wang, X. Q. Senior high school students' understanding about the mathematical function. *JME*, **16**(4): 84-87. T3b

Thienard, J.-C. Redonner du sens aux mathématiques enseignées, *R-IREM*, **66**, 62-72. T1, 5

Thienard, J.-C. Introduction aux lois de probabilités continues: problèmes épistémologiques, *R-IREM*, **67**, 31-42. T2b, 5, 4

Yang, Y. H., Wei, J., & Song, N. Q. An analysis of historical materials in primary mathematics textbooks. *JME*, **16**(4): 80-83. T3b

2008

Bagni, G.T. A theorem and its different proofs: History, mathematics education and the semioticcultural perspective. *Canadian Journal of Science, Mathematics, and Technology Education*, **8**(3), 217–232. T5, 1

Bühler, M. & Michel-Pajus, A. Les démonstrations en arithmétique: à propos de quelques preuves historiques du petit théorème de Fermat. *R-IREM*, **71**, 23-39. T2b, 4

Clark, K.M. & Robson, E. Ancient accounting in the modern mathematics classroom. *BSHM*

23(3), 129-142. T2

Doorman, M. & van Maanen, J. A historical perspective on teaching and learning calculus. *Australian Senior Mathematics Journal*, **22**(2), 4-14. T2

Fried, M.N. History of mathematics in mathematics education: A saussurean perspective. *TMMEM*, **5**(2), 185-198. T1, 2

Gérini, C. Variations pédagogiques sur un article de géométrie analytique d'Haton de la Goupillière, paru en 1872. *R-IREM*, **71**, 65-80. T2b,a, 4

Jahnke, H.N. & Richter, K. Geschichte der Mathematik. Vielfalt der Lebenswelten – Mut zu divergentem Denken. *Mathematik lehren*, **151**, Dez. 2008, 4-11. T1

Jamm, F. Le rêve de Ptolémée réalisé. *R-IREM*,**73**, 5-19. T5, 2a, b, 4

Jankvist, U.T. A teaching module on the history of public-key cryptography and RSA. *BSHM*, **23**(3), 157–168. T2

Jankvist, U.T. Den matematikhistoriske dimension i undervisning—gymnasialt set (The dimension of the history of mathematics in teaching—the case of upper secondary level). *MONA*, **4**(1), 24–45 (in Danish). T2, 1, 4

Zhao, Y. Y., & Zhang, X. M. Discussions about the Historical Parallelism: an Overview. *JME*, **17**(4), 53-56. T1

2009

Charalambus, C., Panaoura, A. & Philippou, G. Using the history of mathematics to induce changes in preservice teachers' beliefs and attitudes: insights from evaluating a teacher education program. *ESM*, **71**(2), 161–180. T2

Henry, M. Emergence de la probabilité et enseignement: définition classique, approche fréquentiste et modélisation. *R-IREM*, **74,** 76-89. T2b, 5, 4

Jankvist 2009b (see section 5). T1

Jankvist, U.T. On empirical research in the field of using history in mathematics education. *Revista Latinoamericana de Investigatión en Matemática Educativa*, **12**(1), 67-101. T1, 2a

Jankvist, U.T. History of modern applied mathematics in mathematics education. *FLM*, **29**(1), 8-13. T2

Kang, S.G. & Hu, G.H. Researches on the history of mathematics and mathematics education in primary and secondary schools in China: Analysis and reflections. *JME*, **18**(5), 65-68. T3b

Kjeldsen, T.H. & Blomhøj, M. Integrating history and philosophy in mathematics education at university level through problem-oriented project work. *ZDM*, **41**, 87–103. T1,2a,5

Lin, M.S. & Su, Y.W. Letting mathematics being interesting by history of mathematics. *The educator monthly*, **509**, 81-83 (in Chinese). T2a

Liu, P-H. History as a platform for developing college students' epistemological beliefs of mathematics. *Int. J. Sci. ME*, **7**(3), 473–499. T2, 1

Mayfield, B. & Kimberly, T. A Locally Compact REU in the History of Mathematics: Involving Undergraduates in Research. *Convergence*, **6**. DOI: 10.4169/loci003263. T2a

Shen, J.L. & Su, Y.W. Integrating history of mathematics into elementary teaching. *New Horizon Bimonthly for Teachers in Taipei*, **163**, 70-77 (in Chinese). T2a

Zhang, X.M. & Wang X.Q. Integrating the History of Mathematics into Mathematics Teaching in Senior High Schools: An Action Research. *JME*, **18**(4), 89-92. T2b

2010

Barbin, É. Epistémologie et histoire dans la formation mathématique. *R-IREM*, **80**, 74-86. T1, 5

Bartolini Bussi, M.G., Taimina D. & Isoda, I. Concrete models and dynamic instruments as early technology tools in classrooms at the dawn of ICMI: from Felix Klein to present applications in mathematics classrooms in different parts of the world. *ZDM*, **42**(1), 19-31. T2

Bressoud, D. M. Historical reflections on teaching trigonometry. *MT*, **104**(2), 106-112. T2b

Clark, K. M. Connecting local history, ancient history, and mathematics: The Eustis Elementary School pilot project. *BSHM*, **25**(3), 132-143. T2

da Conceição Ferreira Reis Fonseca, M. Adult education and ethnomathematics: appropriating results, methods, and principles. *ZDM*, **42**(3), 361-369. T2, 6, 1

Haverhals, N. & Roscoe, M. The history of mathematics as a pedagogical tool: Teaching the Integral of the secant via Mercator's projection. *The Montana Mathematics Enthusiast*, **7**(2), 339-368. T2a, T5

Jankvist, U.T. An empirical study of using history as a 'goal', *ESM* **74**, 53-74. T2a

Lih, K.W. A remarkable Euler square before Euler *Mathematics Magazine* **83**, 163-167. T3b

Lin, F.M. & Horng, W.S. *The Housekeeper and the Professor* by Yoko Ogawa. *Mathematical Intelligencer*, **32**(2), 75-76. T6

Papadopoulos, I. "Reinventing" techniques for the estimation of the area of irregular plane figures: From the eighteenth century to the modern classroom. *Int. J. Sci. ME*, **8**(5), 869-890. T2

Rowlands, S. A pilot study of a cultural-historical approach to teaching geometry. *S&E*, **19**(1), 55–73. T2a

Tan, X. Z. Spiritual heritage and educational value of Greek mathematical culture. *JME*, 19(1): 27-29. T3a

Tatar, E., Tevfik I., Yasin S. & Levent A. Teaching square root numbers: from the perspective of Ibrahim Hakki of Erzurum, *Procedia – Social and Behavioural Sciences*, **2**(2), 1102–1106. T2b

Xu, Z.T., Wang, X.Q. & Mei Q.X. The genetic approach to teaching and learning mathematics: from the history to the classroom. *JME*, **19**(1), 10-12. T1

Yee, L. & Chapman, E. (2010). Using history to enhance student learning and attitudes in Singapore mathematics classrooms. *Education Research and Perspectives*, **37**(2), 110-132. T2

Zhu, F. Q., & Xu, B. H. The modes of integrating the history of mathematics into teaching: an international perspective. *JME*, **19**(3), 22-25. T1

2011

Guillemette, D. (2011). L'histoire dans l'enseignement des mathématiques: sur la méthodologie de recherche. *Petit x*, **86**(1), 5–26. T1

Heeffer, A. Historical objections against the number line. *S&E*, **20**(9), 863–880. T1

Jankvist, U.T. Anchoring students' metaperspective discussion of history in mathematics, *JRME*, **42**(4), 346-385. T1, 2a

Jankvist & Kjeldsen (see section 5). T1, 2a, 5

Klyve, D., Dtemkoski, L. & Tou, E. Teaching and research using original sources from the

Euler archive, *Convergence*, **8**. DOI:10.4169/loci003672. T2b

Panagiotou, E. Using history to teach mathematics: The case of logarithms. *S&E*, **20**, 1–35. T2a

Pu, S.P. & Wang, X.Q. Hans Freudenthal's thought on HPM and its implications. *JME*, **20**(6), 20-24. T1

Shih, Y.K. & Su, Y.W. Integrating digital materials of history of mathematics into elementary teaching. *National Education*, **52**(3), 65-71. T2a

Schubring (see section 5). T1

Su, Y.W. An initial investigation into the design of materials of history of mathematics for elementary school education. *Research and Development in Science Education Quarterly*, **62**, 75-96 (in Chinese). T2b

Wang, X.Q., Wang, M. & Zou, J. C. Teaching mathematics from the HPM perspective: the case of ellipse. *JME*, **20**(5), 20-23. T2b

2012

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Testa, G. Per un avvio alla ricerca "storica" in campo scientifico: studential lavoro. *L' insegnamento della matematica e delle scienze integrate*, **24B**, 39-68. T2a

2002

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2003

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Jahnke, H. N. Texte von Studierenden zur Geschichte der Mathematik. In: L. Hefendehl - Hebeker & S. Hußmann (Eds.), *Mathematikdidaktik zwischen Fachorientierung und Empirie. Festschrift für Norbert Knoche*, Hildesheim: Franzbecker 2003, 105-116. T4

2004

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2007

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Heeffer, A. Learning concepts through the history of mathematics, In K. François, K. & J.P. van Bendegem, (Eds.). *Philosophical Dimensions in Mathematics Education* (pp. 83–103). New York: Springer (accessed 18/2/2016). T5, 1

Pinxten, R. & François, K. Ethnomathematics in practice, In K. François, K. & J.P. van Bendegem, (Eds.). *Philosophical dimensions in mathematics education* (pp. 213-227). New York: Springer (accessed 18/2/2016). T6, 1

2008

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Furinghetti, F. & Radford, L. Contrasts and oblique connections between historical conceptual developments and classroom learning in mathematics. In L. English (Ed.), *Hand Int RME*, 2nd Edition (pp.626–655). New York: Routledge, Taylor and Francis (accessed 18/2/2016). T1, T3a

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Science, vol.11 (pp.479–491). Netherlands: Springer. T2

2011

Jahnke, H.N. The conjoint origin of proof and theoretical physics. In M. Pitici (Ed.), *The Best Writing on Mathematics 2011* (pp.236 – 256). Princeton: Princeton University Press. T2

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Furinghetti (see section 5). T3a, T1

Siu, M.K. Proof in the Western and Eastern traditions: Implications for mathematics education. In G. Hanna, M. de Villiers (Eds.) *Proof and Proving in Mathematics Education: The 19th ICMI Study* (pp.431–440). New York: Springer. T1, 6, 5

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Wang, X.Q. Research on HPM: An overview. In J. S. Bao & B. Y. Xu (Eds.), *Guide to the Research to Mathematical Education II* (pp. 403-422). China: Jiangsu Education Publishing House. T1, 2, 3, 4

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Clark, K.M. History of mathematics in mathematics teacher education. In M.R. Matthews (Ed.) *Int. Hand. Res. Hi. Phil. Sci. Teach.*, Volume 1, (pp.755–791). Dordrecht: Springer. T1, 2

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Rogers, L. History of mathematics in and for the curriculum. In H. Mendick & D. Leslie (Eds.), *Debates in Mathematics Education* (pp.106-122). London: Routledge. T3

2015

Siu, M. K. "*Zhi yì xíng nán* (knowing is easy and doing is difficult)" or vice versa? -A Chinese mathematician's observation on HPM (History and Pedagogy of Mathematics) activities. In B. Sriraman, J.F. Cai, K. Lee, L. Fan, Y. Shimuzu, C. Lim, K. Subramanian (Eds.), *The First*

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2002

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Safuanov, I.S. Design of genetic teaching of algebra at universities. In I. Vakalis, D. Hughes-Hallett, Ch. Kourouniotis, D. Quinney, & C. Tzanakis (Eds.), *Proc. of the 2nd ICTM* (e-book). New York: J. Wiley & Sons Inc. T2b

2004

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2005

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2006

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Schubring (see section 5). T1

2009

Montelle, C. & Clark, K.M. Using history to enrich undergraduate mathematics: Beyond the anecdote. In D. Wessels (Ed.), *Seventh Southern Right Delta Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics* (pp.185-195). Stellenbosch, South Africa: DELTA. T1, 2

2010

Kjeldsen, T.H. History in mathematics education - why bother? Interdisciplinarity, mathematical competence and the learning of mathematics. In B. Sriraman & V. Freiman (Eds.) *Interdisciplinarity for the 21st Century: Proc. of the 3rd International Symposium on Mathematics and its Connections to Arts and Sciences* (pp.17-48). *TMMEM*, vol. 11. Charlotte, NC: Information Age Publishing. T1, 2a, 5

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Rogers, L. (2012). Practical mathematics in 16th century England: Social-Economic contexts and emerging ideologies in the new Common Wealth. In Bjarnadσttir, K., Furinghetti, F., Matos, J. & Schubring, G. (Eds.),"*Dig where you stand" 2. Proc. of the conference on the History of Mathematics Education* (pp.421- 444). Universidade Nova: Lisbon. T6

2013

Barbin, É. L'histoire des mathématiques dans la formation: une perspective historique (1975-2010). In J.-L. Dorier (Ed.) *Actes du Colloque Espace Mathématique Francophone* (pp.546-554). University of Genève: Genève. T2b

2015

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4. Concluding remarks

The *HPM perspective* described in §1.1 emerged gradually over the last decades as a perception of mathematics worth exploring, thanks to research and teaching work done worldwide, thus establishing the *HPM domain* as a valuable research area in the context of ME. Launching the ICMI Study volume in 2000 was a decisive step in this direction. This highly collective work motivated, stimulated, oriented, encouraged and supported research in this area, to a large extent realized in the context of the *HPM Group* and the main activities related to it. At that time central issues were (and still are):

- To put emphasis on *pre- and in-service teacher education* as a necessary prerequisite for the *HPM perspective* to be possible at all.

- To design, produce, make available and disseminate a variety of *didactical source material* in the form of anthologies of original sources, annotated bibliography, description of teaching sequences/modules to serve as a source of inspiration and/or as generic examples for classroom implementation, educational aids of various types, appropriate websites etc.

- To perform systematically, carefully designed and applied *empirical research* in order to examine in detail and *evaluate* convincingly the *effectiveness* of the *HPM perspective* on improving the teaching and learning of mathematics, as well as students and teachers' awareness of mathematics as a discipline and their disposition towards it.

- To acquire a deeper understanding of theoretical ideas put forward in the HPM domain and to carefully develop them into coherent *theoretical frameworks and methodological schemes* that will serve as a foundation for further research and applications in this area.

In the last 10 to 15 years much work has been done on these issues and more is still in progress. In this survey

- An attempt was made to provide enough evidence - mainly based on the literature - that HM is relevant to ME in several ways and may have a multifaceted influence on improving the teaching and learning of and about mathematics. More specifically:
- An outline of the development of the HPM domain has been given;
- The key issues in this domain have been formulated and briefly discussed; and
- A sufficiently comprehensive survey of the existing literature has been included.

We hope that the present survey will serve both as a working document and as a motivation for all those who desire information about the *HPM perspective* and to explore further the possibilities offered for supporting and improving ME.

Acknowledgments: We much appreciate contributions to section 3 from É. Barbin, F. Furinghetti, Y. Hong, W.-S. Horng, N. Jahnke, U.T. Jankvist, M. Moyon, D. Pengelley, L. Rogers, M.-K. Siu, Y.-W. Su, F. Swetz, Y. Thomaidis. They provided detailed references of relevant publications, though lack of space did not allow including all of them here.

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