HPM 2016

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HISTORY OF MATHEMATICS IN MATHEMATICS EDUCATION

Recent Developments

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Dedicated to the memory of

Mustafa Alpaslan

(1988-2015)

Friend and Colleague

Overview of this presentation

- **Aim**, **Focus**, **Rationale**, & **Background** of the survey
- **Structure** of the survey
- **Outline** by section
- **Main purpose**: Keep the survey regularly updated
- **Feedback:** Questions for discussion

Aim

- To survey the **state-of-the-art** of the **HPM domain**
  - approaches adopted towards *integrating History & Epistemology of Mathematics in Mathematics Education*
- To provide **concise annotated reference** to work in this area

Focus

- Account of **developments since 2000**
  - following the publication of the landmark in this domain: J. Fauvel & J. van Maanen (Eds.) *History in Mathematics Education: The ICMI Study*

Survey available online from

- **HPM website**:

  http://www.clab.edc.uoc.gr/HPM/HPMinME-TopicalStudy-18-2-16-NewsletterVersion.pdf

- **HPM 2016 website**:

  http://hpm2016.sciencesconf.org/conference/hpm2016/pages/HPMinME_TopicalStudy_18_2_16_NewsletterVersion.pdf

- ***Proceedings of HPM 2016***

  L. Radford, F. Furinghetti, T. Hausberger (Eds.) pp. 135-179

Rationale

The ***HPM*** perspective: ***H*** + ***P*** + ***M***

The **fruitful synthesis** of

- ***H***: the **non-absolute** nature of human **knowledge**
- ***P***: Understanding/appreciating **differences** among **humans**
- ***M***: Logical/ rational/ intellectual **rigor** & **consistency**

This **dialectical interplay** is the special **characteristic** of the ***HPM perspective*** that contributes towards making ***Mathematics*** **understandable**, hence, **accessible** & **acceptable**

Background

An ICME-13 pre-conference document on research related to:

***Themes** of the*

Topic Study Group 25: ***The Role of History of Mathematics in Mathematics Education:***

1. Theoretical and/or conceptual frameworks for integrating history in ME
2. History and epistemology implemented in ME, considered from either the cognitive or/and affective points of view:
    a) Classroom experiments at school, the university and teacher pre- & in-service education
    b) Teaching material: textbooks, resource material of any kind

Background, *continued*

An ICME-13 pre-conference document on research related to:

***Themes** of the*

Topic Study Group 25: ***The Role of History of Mathematics in Mathematics Education** (continued):*

3. Surveys on:
   a) Research on the HM in ME
   b) The HM as it appears in curriculum and/or textbooks
4. Original sources in the classroom and their educational effects
5. History and epistemology as a tool for an interdisciplinary approach in the teaching and learning of mathematics and the sciences by unfolding their fruitful interrelations
6. Cultures and mathematics fruitfully interwoven

Document structured in five (5) sections

1. *The **HPM Perspective***
2. *Outline of the **historical development** of the **HPM domain***
3. *A **bibliographical survey** in the HPM domain **since 2000***
4. ***Summary** of the main points of the survey*
5. *Bibliography for section 2*

Section 1: *The* ***HPM Perspective***

- Mathematics is a ***human intellectual enterprise*** with a long ***history*** and a ***vivid present***.
- Mathematical knowledge is determined **both** by
  - the circumstances in which it becomes a ***deductively structured corpus*** (the "polished products" of mathematical activity)
  - the ***procedures*** that originally led/may lead to it (implicit motivations, sense-making actions, reflective processes, aiming to the construction of meaning).
- *Perceiving Mathematics* **both** *as*
  - ***a logically structured collection*** of ***intellectual products*** &
  - ***processes*** of ***knowledge production***

should be the **core** of its **teaching** and central to its image communicated to the outside world.

Section 1: *The **HPM Perspective** (continued)*

• Integrating ***historical*** & ***epistemological*** issues in mathematics ***teaching*** & ***learning***

- a way for ***exposing mathematics in the making***,
- opportunities for better ***understandings of mathematics***,
- deeper ***awareness*** of ***mathematics*** as a ***discipline***

• Exploring the multifarious role of ***History*** & ***Epistemology*** of *Mathematics* in ***Mathematics Education*** along these lines is

- an interesting perspective with a long history as a research domain in ME (the ***HPM perspective***),
- the central theme and concern of the ***HPM Group*** since its formation (1972, at ICME 2)

Section 2: *Outline of the **historical development** of the **HPM domain***

- **Brief account of main activities and their outcomes since 2000**

  - *Early history* (meetings, publications, formation of local (sub)groups…)

  - *European Summer University on the History and Epistemology in Mathematics Education* (ESU) since 1993

  - Short presentation of journals & newsletters (*HPM Newsletter*, *Convergence*, *BSHM Bulletin*)

  - Activities within *ICME* since 2000

  - Activities within *CERME* since 2009

Section 2: *Outline of the **historical development** of the **HPM domain** (continued)*

- Outline & comments on **key issues** explicitly addressed in and/or implicitly underlying the ***HPM perspective***
  In connection with **objections** against this perspective

- ***Which history*** *is suitable, pertinent, and relevant to Mathematics Education (ME)?*
- ***Which role*** *can History of Mathematics (HM) play in ME*
- *To **what extent** has **HM** been **integrated in ME** (curricula, textbooks, educational aids/resource material, teacher education)?*
- ***How*** *can **this role** be **evaluated** and **assessed** and to what extent does it contribute to the teaching and learning of mathematics?*

Section 3: *A bibliographical survey in the HPM domain since 2000*

The major part of the survey. **Comprehensive bibliography** of work related to the Themes of TSG 25 of ICME-13:

3.1 Collective works devoted to the HPM perspective:

- *Proceedings of conferences and meetings*
- *Collective volumes*
- *Special issues* of international journals of ME
- *Resource material*, collectively produced, especially material available on the web

3.2 Individual works:

- *Books & doctoral dissertations*
- *Papers in scientific journals*
- *Papers in collective volumes*
- *Papers in conference proceedings*

Section 3: *A bibliographical survey in the HPM domain since 2000 (continued)*

Remarks:

a) In each subsection, **items** classified by **publication year**; for each year classified by authors' alphabetical order

b) For **each item** the **TSG 25 themes** related to it are **indicated**

c) Emphasis on research results of international interest; but due attention to nationally-oriented implementations as well

d) Collective works exclusively on the *HPM perspective* **annotated** briefly. To keep the survey to a reasonable size, contributions there, **not** included as **separate** items in § 3.2.

e) Lengthy **URL** integrated **into each reference's title**

(Advice: **use** the **regularly updated electronic** version from the ***HPM*** website)

a) Survey **continually updated** & **available online**

Section 3: *A bibliographical survey in the HPM domain since 2000 (continued)*

Examples

3.1.1 ***Collective volumes*** with research papers, reviews of work …

Barbin, É. (Ed.). *Des grands défis mathématiques d'Euclide à Condorcet*. Paris: Vuibert. (2010). **T2, 4, 5, 1**

9 examples of introducing a historical perspective in ME, having as a starting point specific historical problems and organized in 4 parts: measuring magnitudes, representing magnitudes, calculating the probable, approaching a curve; thus illustrating the great domains of today's taught math: analysis, algebra, probability and geometry (for details see http://culturemath.ens.fr/node/2582 (accessed 18/2/2016) or *HPM Newsletter* No75/2010, pp. 1-2).

Section 3: *A bibliographical survey in the HPM domain since 2000 (continued)*

Examples

3.1.2 ***Special issues*** of international journals of ME

Furinghetti, F., Radford, L., & Katz, V. (Eds.). The history of mathematics in mathematics education: Theory and practice. *ESM*, *66*(2), 107-271. (2007). **T1, 2a, 3b, 4**

A special issue with 10 papers, seeking to deepen the understanding of the pedagogical role HM may play in contemporary ME. Some provide examples of the use of the HM in school practice and teacher education; others address theoretical questions that have become crucial to understanding the profound intertwining of past and present, conceptual developments on spreading new epistemologies and theories of learning.

Section 3: *A bibliographical survey in the HPM domain since 2000 (continued)*

Examples

3.1.3 ***Proceedings of conferences and meetings*** (with reference to their accessibility via the Internet, wherever possible)

Jankvist, U. T., Chorlay, R., & Clark, K., Lawrence, S., & van Maanen, J. (Eds.). WG 12: History in Mathematics Education. In K. Krainer & N. Vondrová (Eds.), *Proc. of CERME 9* (pp. 1778-1884). Prague: Charles University in Prague, Faculty of Education & ERME (2015). (accessed 4/4/2016).

Group work structured along 9 themes leading to 14 papers and 2 posters included in this proceedings, dealing with four areas of questions on history in ME-the student perspective; history in ME-the teacher perspective; history of ME-the mathematical education landscape; methodological reflections on history in/of ME.

Section 3: *A bibliographical survey in the HPM domain since 2000 (continued)*

Examples

3.1.4 ***Resource material***, collectively produced, especially material available on the web.

Barnett, J., Bezhanishvili, G., Leung, H., Lodder, J., Pengelley, D., Pivkina, I., Ranjan, D., & Zack, M. Primary historical sources in the classroom: discrete mathematics and computer science. *Convergence*, *10*. (2013). doi:10.4169/loci003984 **T2b**

Presentation of 16 separate curricular modules, each a project for students based on excerpts from primary historical sources, to provide context, motivation and direction for selected topics in discrete mathematics and computer science as an alternative form of instruction.

Section 4: *Summary of the main points of the survey*

Central issues for future research within the *HPM domain*

- To put emphasis on *pre- and in-service teacher education* as a necessary prerequisite for the *HPM perspective* to be possible
- To design, produce, make available and disseminate a variety of *didactical source material*
- To perform systematically *empirical research*, carefully designed and applied for detailed *evaluation* of the *effectiveness* of the *HPM perspective*
- To develop theoretical ideas into coherent *theoretical frameworks* and *methodological schemes*

Section 5: *Bibliography for section 2*

This Survey

should be considered **only** as a **working document to be kept continuously updated**

to **amend** this document

- **make comments**;
- **point out omissions**;
- **suggest corrections**;

and especially

- **provide further bibliographical references**

Your contribution is most desirable & welcome!

Some questions

1(a). Section 2 raises **four** issues arguing that these have always been the **main issues** underlying **research** in the **HPM domain**. Do you agree?

1(b). Do you think that there are **other** issues of **equal significance** independent of these four issues?

Section 2: **Key issues** explicitly addressed in and/or implicitly underlying the ***HPM perspective***

- ***Which history*** *is suitable, pertinent, and relevant to Mathematics Education (ME)?*
- ***Which role*** *can History of Mathematics (HM) play in ME*
- *To* ***what extent*** *has* ***HM*** *been* ***integrated in ME*** *(curricula, textbooks, educational aids/resource material, teacher education)?*
- ***How*** *can* ***this role*** *be* ***evaluated*** *and* ***assessed*** *and to what extent does it contribute to the teaching and learning of mathematics?*

Some questions

2. In section 2 a number of **objections** **against** an **HPM perspective** in education are presented.

Which ones (if any) do you think **point to** the **most grave difficulties** that are **difficult**

- to **overcome/deal** with, and/or
- have **not been considered** effectively, so far?

Some questions

A *Objections of an epistemological and methodological nature*

(a) *On the nature of mathematics*

1. This is **not mathematics**! Teach the subject first; then its history.

2. Progress in mathematics is to make difficult problems routine, so **why bother to look back**?

3. What really happened can be rather tortuous. Telling it as it was can **confuse rather than enlighten**!

(b) *On the difficulties inherent to this approach*

1. Does it really help to read **original texts**, which is a very **difficult** and **time-consuming** task?

2. Is it liable to breed **cultural chauvinism** and parochial **nationalism**?

3. **Students** may have an **erratic sense** of the past that makes **historical contextualization** of mathematics **impossible** without having a broader education in general history.

Some questions

B *Objections of a practical and didactical nature*

(a) *The background and attitude of teachers*

1. Lack of didactical time: **no time** for it **in class**!
2. **Teachers** should be **well educated in history**: “I am not a professional historian of mathematics. How can I be sure of the exposition’s accuracy?”
3. **Lack** of **teacher training**.
4. **Lack** of appropriate didactical and **resource material**.

(b) *The background and attitude of the students*

1. They regard it as history and they **dislike history** class!
2. They regard it just as **boring as mathematics** itself.
3. They do **not** have enough general **knowledge of culture** to appreciate it.

(c) *Assessment issues*

1. **How** can you set questions on it in a **test or exam**?
2. Is there any **empirical evidence** that students **learn better** when HM is made use of in the classroom?

Some questions

3(a). Section 4 states **four** issues as **most important** for future **research** in the **HPM domain**. Do you agree?

3(b). Do you think that there are **other** issues of **equal significance** independent of these four issues?

Section 4: *Central issues for future research in the HPM domain*

- To put emphasis on ***pre- and in-service teacher education*** as a **necessary prerequisite** for the *HPM perspective* to be possible
- To design, produce, make available and disseminate a variety of ***didactical source material***
- To perform systematically ***empirical research***, carefully designed and applied for detailed ***evaluation*** of the ***effectiveness*** of the *HPM perspective*
- To develop theoretical ideas into coherent ***theoretical frameworks*** and ***methodological schemes***

Some questions

4. Any **ideas** for **improving** the structure & content of the **bibliographical survey** of section 3?

5. Any **other comments**?

THANK YOU

for your attention